INTERNATIONAL CONFERENCE (ONLINE)
ON
NUMBER THEORY AND DISCRETE MATHEMATICS (ICNTDM)

11-DECEMBER-2020 to 14-DECEMBER-2020

"To mark hundredth year of passing away of Srinivasa Ramanujan"

Organised by
RAMANUJAN MATHEMATICAL SOCIETY
Hosted by
RAJAGIRI SCHOOL OF ENGINEERING & TECHNOLOGY (RSET) - AUTONOMOUS, Kochi, India

https://www.rajagiritech.ac.in/icntdm/

ICNTDM 2020

The Ramanujan Mathematical Society (RMS), will be organising ICNTDM-2020 to mark the 100th year of premature passing away of the legendary Indian mathematician Srinivasa Ramanujan (22nd December 1887-26th April 1920).
All aspects of Mathematics influenced by Ramanujan and in particular Number Theory – a subject which had its historic root in India, and Discrete Mathematics will be discussed in this conference.

Conference Themes

- Number Theory and related areas
- Discrete Mathematics and related areas

Academic Sessions for Conference

- Plenary talks
- Invited talks
- Contributory papers

Registration

- Register online at https://forms.gle/23g1wKd9e28MGFDAA
- Registration begins: 25 August 2020
- Registration ends: 1 December 2020

Call for Papers

Original research papers in full can be submitted for presentation in the contributory paper session
Full paper shall be submitted through Easy Chair:
https://easychair.org/conferences/?conf=icntdm2020
Papers should be on
- Number theory
- Discrete Mathematics

Contributory Papers

- Paper submission begins: 1st September 2020
- Paper submission ends: 1st November 2020

Proceedings

- Proceedings will be published as a special issue of scopus indexed journal, JOURNAL OF RAMANUJAN MATHEMATICAL SOCIETY.

International Academic Programme Committee

- Alladi Krishnaswami, University of Florida, USA
- Bimal Roy, Indian Statistical Institute, India
- Carsten Thomassen, Technical University of Denmark, Denmark
- Dhruv Mubayi, University of Illinois, USA
- George Andrews, Pennsylvania State University,USA
- Gyula O. H. Katona, Hungarian Academy of Sciences, Alfréd Rényi Institute of Mathematics, Hungary
- Maria Chudnovsky, Princeton University, USA
- Michel Waldschmidt, Sorbonne Université,Institut Mathématique de Jussieu,FRANCE
- Pavol Hell, Simon Fraser University, Canada
- R Balasubramanian, National Centre for Mathematics, Indian Institute of Technology, Mumbai, India
- R.B. Bapat, Indian Statistical Institute, India
- Rekha R Thomas, University of Washington, USA
- Sandi Klavžar, University of Ljubljana, Slovenia
- Tetali Prasad, Georgia Institute of Technology, USA
- Arvind V, Institute of Mathematical Sciences,India
- Ambat Vijayakumar, Cochin University of Science & Technology, India - Chair

Advisory Committee

Patrons:

Rev. Fr. Benny Nalkara CMI , Manager, Rajagiri School of Engineering & Technology, India
Rev. Fr. Dr. Jose Kuriedath CMI ,Director, Rajagiri School of Engineering & Technology, India
S. Ponnusamy, IIT Madras, President Ramanujan Mathematical Society, India

Members:

C.S. Aravinda, TIFRCAM, Bangalore, Secretary, Ramanujan Mathematical Society, India
E. Sampathkumar, Mysore University, India
K. Poulose Jacob, Dean Research, Rajagiri School of Engineering & Technology, India
Kuldeep Singh Charak, Jammu University, Academic Secretary, Ramanujan Mathematical Society, India
P.S. Sreejith, Principal, Rajagiri School of Engineering & Technology, India
R. Balakrishnan, Bharathidasan University, India
R. Sivaramakrishnan, Retd. Professor, University of Calicut, India
Saber Elaydi, Trinity University, USA
T.Thrivikraman, Retd. Professor, Cochin University of Science & Technology, India
V. Kannan, S R M University, India

DEPARTMENT OF MATHEMATICS
RAJAGIRI
SCHOOL OF ENGINEERING & TECHNOLOGY (AUTONOMOUS)
Rajagiri Valley, Kochi-682 039, Kerala, INDIA
Tel: +91 484 2660999| Fax: +91 484 2426241
www.rajagiritech.ac.in/icntdm

Contact Us

Conference mail: icntdm2020@rajagiritech.edu.in

Ambat Vijayakumar, Cochin University of Science & Technology (Vice President, Ramanujan Mathematical Society)
Chair, IAPC Email: vambat@gmail.com

Vinod Kumar P B, Rajagiri School of Engineering & Technology
Convenor Email: vinod_kumar@rajagiritech.edu.in

INTERNATIONAL CONFERENCE
ON
NUMBER THEORY AND DISCRETE MATHEMATICS (ICNTDM)

11-DECEMBER-2020 to 14-DECEMBER-2020

"To mark hundredth year of passing away of Srinivasa Ramanujan"

Organised by

RAMANUJAN MATHEMATICAL SOCIETY

Hosted by

RAJAGIRI SCHOOL OF ENGINEERING & TECHNOLOGY (RSET) - AUTONOMOUS, Kochi, India

https://www.rajagiritech.ac.in/icntdm/

RSET RAJAGIRI SCHOOL OF ENGINEERING & TECHNOLOGY AUTONOMOUS

RAMANUJAN MATHEMATICAL SOCIETY PROMOTE PERCEIVE PRESERVE FOUNDED 1985

Plenary Speakers

Alladi Krishnaswami
University of Florida, USA

Arvind V
Institute of Mathematical Sciences,India

Bojan Mohar
Simon Fraser University, Canada

Dhruv Mubayi
University of Illinois, USA

Douglas B West
University of Illinois, USA

George Andrews
Pennsylvania State University,USA

Gyula O. H. Katona
Hungarian Academy of Sciences, Alfréd Rényi Institute of Mathematics, Hungary

Janos Pach
Alfréd Rényi Institute of Mathematics, Hungary

Kannan Soundararajan
Stanford University,USA

Ken-ichi Kawarabayashi
National Institute of Informatics, Japan

Kohji Matsumoto
Nagoya University, Japan

Michel Waldschmidt
Sorbonne Université,Institut Mathématique de Jussieu,FRANCE

Neal Koblitz
University of Washington, USA

Nitin Saxena
Indian Institute of Technology, Kanpur, India

Peter J Cameron
University of St. Andrews, UK

Peter Sarnak
Institute of Advanced Study, USA

Prasad V Tetali
Georgia Tech., USA

R Balasubramanian
National Centre for Mathematics, Indian Institute of Technology, Mumbai, India

Rekha R Thomas
University of Washington, USA

Ritabrata Munshi
Indian Statistical Institute, Calcutta, India

Sandi Klavžar
University of Ljubljana, Slovenia

V Kumar Murty
University of Toronto, Canada

Yuri Nesterenko
Moscow State University, Russia

Invited Speakers

B Sury
Indian Statistical Institute, Bangalore, India

Frank Thorne
University of South Carolina, USA

Kalyan Chakraborty
Harish-Chandra Research Institute,India
Kerala School of Mathematics, India

Mahesh Kakde
Indian Institute of Science, India

Martin Milanic
University of Primorska, Slovenia

Mathias Schacht
University of Hamburg, Germany

Oswin Aichholzer
Graz University of Technology, Austria

Sanoli Gun
institute of Mathematical Sciences, India

Sebastian Cioaba
University of DeLaware, USA

Stephan Wagner
Uppsala University, Sweden

Venkatesh Raman
Institute of Mathematical Sciences, India

Yongtang Shi
Nankai University, China

IMPORTANT DATES

- Registration Begins: 25 August 2020
- Registration Ends: 1 December 2020
- Paper Submission for Contributory Session Begins: 1 September 2020
- Paper Submission for Contributory Session Ends: 1 November 2020
- Notification of acceptance of papers: 15 November 2020
- Conference: 11-14 December 2020

DEPARTMENT OF MATHEMATICS
RAJAGIRI
SCHOOL OF ENGINEERING & TECHNOLOGY (AUTONOMOUS)
Rajagiri Valley, Kochi-682 039, Kerala, INDIA
Tel: +91 484 2660999| Fax: +91 484 2426241
www.rajagiritech.ac.in/icntdm

Contact Us

Conference mail: icntdm2020@rajagiritech.edu.in

Ambat Vijayakumar, Cochin University of Science & Technology (Vice President, Ramanujan Mathematical Society)
Chair, IAPC Email: vambat@gmail.com

Vinod Kumar P B, Rajagiri School of Engineering & Technology
Convenor Email: vinod_kumar@rajagiritech.edu.in

International Conference on Number Theory and Discrete Mathematics (ONLINE)

(ICNTDM)

11 - 14, DECEMBER-2020

Organised by

RAMANUJAN MATHEMATICAL SOCIETY

Hosted by

DEPARTMENT OF MATHEMATICS

RAJAGIRI SCHOOL OF ENGINEERING & TECHNOLOGY

(AUTONOMOUS)

Kochi, India

PREFACE

The International Conference on Number Theory and Discrete Mathematics (ICNTDM), organised by the Ramanujan Mathematical Society (RMS), and hosted by the Rajagiri School of Engineering and Technology (RSET), Cochin is a tribute to the legendary Indian mathematician Srinivasa Ramanujan who prematurely passed away a hundred years ago on 26th April 1920 leaving a lasting legacy.

Conceived as any usual conference as early as June 2019, the ICNTDM was compelled to switch to online mode due to the pandemic, like most events round the globe this year.

The International Academic Programme Committee, consisting of distinguished mathematicians from India and abroad, lent us a helping hand every possible way, in all aspects of the conference. As a result, we were able to evolve a strong line up of reputed speakers from India, Austria, Canada, China, France, Hungary, Japan, Russia, Slovenia, UK and USA, giving 23 plenary talks and 11 invited talks, in addition to the 39 speakers in the contributed session.

This booklet consists of abstracts of all these talks and two invited articles; one describing the various academic activities that RMS is engaged in, tracing history from its momentous beginning in 1985, and a second article on Srinivasa Ramanujan. It is very appropriate to record here that, in conjunction with the pronouncement of the year 2012 – the 125th birth anniversary year of Ramanujan – as the ‘National Mathematics Year’, RMS heralded an excellent initiative of translating the celebrated book ‘The Man Who Knew Infinity’ into many Indian languages. Leading publishers like the National Book Trust and the Kerala Language Institute joined hands with the RMS in successfully realising this unprecedented visionary initiative.

We thank the Organising Committee, the Advisory Committee and the International Academic Programme Committee for their unstinting help at various stages of the conference.

We fondly hope that this conference will initiate research collaborations among the participants from different countries.

We heartily welcome you all to this event, and wish you a vibrant academic deliberation in the ICNTDM.

WISH YOU ALL A HEALTHY, PROSPEROUS NEW YEAR - 2021.

Ambat Vijayakumar
(Vice President, RMS)
Chair- IAPC , ICNTDM
Cochin University of Science
and Technology
10th December 2020

P.B.Vinodkumar
Convener, ICNTDM
Rajagiri School of Engineering
and Technology

INTERNATIONAL CONFERENCE ON NUMBER THEORY AND DISCRETE MATHEMATICS

ICNTDM 2020

11-14, DECEMBER 2020

You are cordially invited to attend the

OPENING CEREMONY on 10th December 2020 at 19.30 hrs. IST

19.30 – 19.33:	Prayer
19.33 – 19.36:	Welcome Address - Prof. P. S. Sreejith, Principal, Rajagiri School of Engineering & Technology (RSET), Cochin, India
19.36 – 19.41:	Address by Prof. R. Balasubramanian, The Institute of Mathematical Sciences, Chennai, India
19.41 – 19.48:	Presidential Address – Rev. Fr. Dr. Jose Kuriedath, Director, RSET, Cochin, India
19.48 – 19.55:	About ICNTDM – Prof. Ambat Vijayakumar, Chair IAPC, ICNTDM, Vice President, RMS
19.55 – 20.15:	Inaugural Address – Prof. Bruce C. Berndt, University of Illinois, USA
20.15 – 20.20:	Felicitation by Prof. S. Ponnusamy, President RMS
20.21 – 20.25:	Felicitation by Prof. R. Balakrishnan, Bharathidasan University, India
20.25 – 20.30:	Felicitation by Prof. Poulose Jacob, Dean Research, RSET, Cochin, India
20.30 – 20.33:	Vote of Thanks – Prof. Vinod Kumar P. B., Convenor ICNTDM
20.33 :	National Anthem

RSET

AUTONOMOUS

INTERNATIONAL CONFERENCE ON NUMBER THEORY AND DISCRETE MATHEMATICS
ICNTDM 2020

11-14, DECEMBER 2020

You are cordially invited to attend the

PUBLIC LECTURE

On

RAMANUJAN, NUMBER THEORY AND PANDEMIC

By

PROFESSOR M. RAMMURTHY FRSC

Queen's University, Canada

On

11th December 2020

at

8 pm IST

Rajagiri School of Engineering & Technology,Rajagiri Valley, Kakkanad, Kochi-682039, Kerala, INDIA
Tel: +91 484 2660999 | Fax: +91 484 2426241 | www.rajagiritech.ac.in | office@rajagiritech.edu.in

Contents

Ramanujan Mathematical Society: An overview

C.S. Aravinda

Secretary RMS, TIFR Centre for Applicable Mathematics, Bangalore, India

Genesis of the Ramanujan Mathematical Society

The Ramanujan Mathematical Society (RMS) was founded in 1985 with the ostensible purpose of promoting mathematics at all levels in the country.

The actual beginning could be traced to the 50th annual conference of the Indian Mathematical Society held in December 1984 at the Sardar Patel University, Anand, Gujarat. On the second day of the conference, (Late) K.S. Padmanabhan, then Director of the Ramanujan Institute for Advanced Study in Mathematics, Chennai gathered some of the participants of the conference and stressed on his view that the country of our size could think of yet another mathematics journal that upkeeps both quality and regularity. It was pointed out by E. Sampathkumar and agreed upon by the rest of the group, that to start such a journal, a mathematics society is required as a launching pad. The matter was deliberated in detail and pursued in depth in the following three months.

Finally, in April 1985, a Promoters' Meeting was convened by R. Balakrishnan at the National College Campus, Tiruchirappalli, Tamil Nadu, when the move fructified, and RMS came into existence. G. Shankaranarayanan was chosen as the first President, R. Balakrishnan as the first Secretary and E. Sampathkumar as the first Academic Secretary. The bye-laws were framed and the Ramanujan Mathematical Society was officially registered at Tiruchirappalli on 15th July 1985.

The activities of the RMS may be divided into two broad categories – publications and academic meetings that include workshops, conferences and training programmes organised by RMS solely or jointly with resonant academic organisations. Here is a quick overview of the two component activities.

Publications of the RMS

a) Journal of the Ramanujan Mathematical Society

With the formation of RMS in 1985, the starting of the Journal of the Ramanujan Mathematical Society (JRMS), which was in the offing, was realised in 1986. As one who mooted the idea, K.S. Padmanabhan, took charge as the Chief Editor of JRMS. During the five year period of his stint at the helm, Padmanabhan laid a firm foundation, nurturing a vision that JRMS should shape into a high quality journal of international repute, and a signal forum for mathematicians of the country to showcase their research.

JRMS further prospered under the stewardship of subsequent chief editors V. Kannan, V. Kumar Murty, R. Parimala and R. Sujatha who worked to foster JRMS as one of the most sought after journals for practitioners, especially from India.

The quarterly issues of JRMS are published with punctual regularity by the untiring efforts of Sampathkumar – the main force behind the scene as Managing Editor – who built the all important infrastructural mechanism to manage the work efficiently.

Perpetuating the wishes of its founding members, JRMS is committed to the upkeep of high academic standards, and the upward trend continues to flourish under the chief editorship of R. Munshi and the team of serious practitioners serving on its editorial board.

b) Mathematics Newsletter

During 1991, the National Board for Higher Mathematics (NBHM) organised a conference on 'Development of Mathematics' at BARC, Mumbai. At this conference, a suggestion was made to start a mathematics newsletter to cater to the needs of mathematics teachers, research scholars and students. While accepting the suggestion, NBHM wanted RMS to run the newsletter. R. Balakrishnan was its Chief Editor from 1991 (the year of starting) until 2002. Currently, S. Ponnusamy of IIT Madras is the Chief Editor.

The Mathematics Newsletter (MNL) publishes expository articles highlighting the latest achievements in the field of mathematics, news of events relevant to mathematics like upcoming mathematics workshops, conferences and exhibitions, international initiatives related to mathematics and things that come under the purview of outreach.

MNL is a quarterly journal published in March, June, September and December of each year. The first issue of any new volume is published in June. It is currently being mailed free to around 1800 institutions throughout India.

c) Lecture Notes and Collected Works Series

The Lecture Notes Series publishes proceedings resulting from a variety of mathematical meetings such as workshops, symposia, seminars and conferences which discuss and report on important developments in Mathematics.

The focus of this series is to foster dissemination and easy availability of a wide spectrum of mathematical literature arising out of meetings ranging from instructional conferences of a pedagogical nature to topical meetings seriously discussing a recent progress of far-reaching impact on the subject. The series also includes expositions of frontline topics for which there are no readily available text books.

Under the Collected Works Series of the RMS, seminal lifetime contributions of eminent and prolific Indian mathematicians whose works have had a telling impact on the subject are brought together.

V. Kumar Murty is the present Chief Editor of this series.

Academic meetings

a) Annual Conferences

RMS Annual conferences are held every year in different locations across the country, typically hosted by a local institution in the venue of the conference. The annual General Body meeting of the RMS is usually on the second day of the annual conferences during which most deliberations, including formally ratifying the memberships of the applications received during the year prior to the conduct of the conference takes place.

b) Compact Courses

RMS organises 'Compact Courses' for postgraduate students in mathematics to collaborate with universities / institutions to teach their students and guide their teachers in some topics in which they need external assistance.

The lectures in a compact course (CC) are normally held over two weeks, and are intended to supplement the lecture program in a specific topic taught in a department. Problem-solving is an important component of this programme.

Compact Courses programme is intended not only to students but also younger mathematicians involved in teaching.

c) APU-RMS Teacher Training Programme

The aim of this programme is to hold workshops at selected schools in different cities across India. The main purpose of these workshops is to create viable platforms where teachers (and through them children) from schools across the country can share their original observations and discoveries and where beautiful results in school level mathematics can be shared, written about and discussed. Each workshop will consist of 30 high school teachers.

d) Undergraduate Teachers Enrichment (UGTE) Programmes

The UGTE programmes are meant for all interested undergraduate faculty across India. The idea here is to form a large mathematics discussion group dealing with aspects of undergraduate mathematics, in the colleges affiliated to universities. This includes statistics, operational research and computer science departments and Engineering colleges as well.

In addition, RMS also organises Memorial Lectures.

Becoming a member of the RMS

Membership of the RMS is available to the academic staff of educational institutions in India who are engaged in teaching and research in mathematics in its broadest interpretation, or to those who hold a degree in mathematics, or who use mathematics significantly in their work, or whose work would benefit from the activities of RMS, or who are interested in promoting and extending the use of mathematical knowledge, or who have a deep commitment to mathematics and an abiding interest in mathematical research.

RMS membership is also available to those who are members of overseas Mathematical Societies and who are not normally resident in India.

Funding for RMS

RMS is fortunate to have received major grants from governmental organisations and several significant donations over the years from kind and generous souls. They have been extremely valuable in enabling RMS conduct its various programmes across the country, as well as help meet some of its publishing costs thereby extending its support for mathematics and mathematicians in India. RMS is immensely grateful for the grants and donations it has received, and humbly accepts any donations from its patrons, benefactors and well-wishers.

For more details, please visit `https://www.ramanujanmathsociety.org/`

About RSET

Rajagiri School of Engineering & Technology (RSET), established in 2001, is a private self-financing college, which is affiliated to the A.P.J. Abdul Kalam Technological University, Trivandrum, Kerala, India. The college is approved by the All India Council for Technical Education, New Delhi. Institution offers 8 UG programmes, 5 PG progarmmes and Ph.D progarmmes in 5 disciplines.

B.Tech. Programmes

- Applied Electronics & Instrumentation Engineering
- Artificial Intelligence & Data Science
- Civil Engineering
- Computer Science & Engineering
- Electrical & Electronics Engineering
- Electronics & Communication Engineering
- Information Technology
- Mechanical Engineering

M.Tech. Programmes

- Communication Engineering
- Computer Science & Information Systems

- Industrial Drives & Control
- Network Engineering
- VLSI & Embedded System

Ph.D Programmes

- Computer Science & Engineering
- Electrical & Electronics Engineering
- Electronics & Communication Engineering
- Information Technology
- Mathematics
- Mechanical Engineering

The Institution is ranked in NIRF and is reaccredited by NAAC with A grade in second cycle Five UG programmes, i.e. B.Tech. Applied Electronics & Instrumentation, B.Tech. Computer Science & Engineering, B.Tech. Electrical & Electronics Engineering, B.Tech. Electronics & Communication Engineering, and B.Tech. Information Technology, have been granted provisional accreditation by NBA for 3 years.

RSET is an endeavour of the Sacred Heart Province of the Carmelites of Mary Immaculate (CMI) - the first-ever indigenous religious congregation for men in the Syrian Catholic tradition of Christianity in India. The CMI congregation, canonically established in 1855, played a major role in revolutionizing the education scene in Kerala in the late 19th century. Heirs to the profound vision of St. Kuriakose Elias Chavara, one of the founding fathers of the congregation, the CMIs' have zealously worked towards providing value-based quality education to society at large, irrespective of religious differences, down the centuries. Today, the congregation has more than 400 educational institutions, from schools to professional colleges under its umbrella.

The Rajagiri Valley campus is a perfect blend of the urban and the rural: while the verdant and serene backdrop of the self-contained campus provides the ideal atmosphere for stimulating the creativity and intellectual pursuits of the academia, the close proximity to the industrial belt of Kerala, the Infopark, and the proposed Smart City helps in providing the students with exposure to the practical aspects of their profession. This ensures a smooth transition from the institution to the industry for our graduates.

Department of Mathematics

The department offers various Mathematics courses in the B.Tech and M.Tech programmes, apart from offering the Ph.D programme in Mathematics. The Department of Mathematics is recognized as a place of research in Mathematics under APJ Abdul Kalam Technological University, Kerala. The Center for Topology and Applications (CETA) established under the department to promote research in Topology and related areas.

Srinivasa Ramanujan : A Self-taught Genius of Inexplicable Originality - Transcending Kanigel's Canvas

Parthasarathi Mukhopadhyay

Ramakrishna Mission Residential College (Autn.), West Bengal, India

G.H. Hardy (1877-1947), FRS and Sadleirian Professor of Pure Mathematics at Cambridge University, Ramanujan's mentor in England who had interacted with Ramanujan's peerless and raw mathematical talent from the closest proximity during his Cambridge days, once told "*It was his insight into algebraic formulae, transformation of infinite series, and so forth, that was most amazing. I have never met his equal, and I can compare him only with Euler or Jacobi...He was by far the greatest formalist of his time....one gift it has which no one can deny— profound and invincible originality...He would probably have been a greater mathematician if he could have been caught and tamed a little in his youth. On the other hand he would have been less of a Ramanujan, and more of a European professor, and the loss might have been greater than the gain.*". In 2020, as we commemorate the Death Centenary of this self-taught legendary son of our soil, the present article is meant to be an humble homage to his unsurpassed tantalizing mathematical skills, that has left the world of mathematics mesmerized with sheer awe for the last hundred years. His typically original ways of doing mathematics, his mathematical thought process has not yet been totally deciphered and his tragic untimely demise, when he was at the peak of his creative genius despite his irrecoverable illness, leaves us to wonder what could have been his further mathematical achievements, had the destiny granted him a longer life!

During his last days when he was only 32 years old, fragile with irrecoverably ill health but still working passionately with full vigor on what will later come to be known as the Mock Theta Function, one of his seminal contributions to the world of mathematics, Ramanujan once told his wife Janaki that his name "*will live for one hundred years*". How amazingly right he was ! On this year of his Death Centenary, his name is referred to over 400 research articles in Mathematics, as a quick search in MathSciNet reveals, of which 11 are from 2020 itself. This numeral arguably may not be proportional to his greatness, but this may still be regarded as some measure to judge the relevance of his beautiful mathematics even in the present century. Prof. Ken Ono, a noted Ramanujan scholar from Emory University points out that "*The legend of Ramanujan has continued to grow with the ever-increasing importance of his mathematics... Clearly, Ramanujan was a great anticipator. His work provided examples of deeper structures, and suggested important questions which are now inescapable in the panorama of modern number theory... his untimely death and the enigmatic nature of his writings resulted in a great mystery. We will never know how he came up with the mock theta functions. We certainly cannot pretend to know what he fully intended to do with them. However, it is clear that he understood that the mock theta functions would go on to play important roles in number theory.*"

Biographies of mathematicians, barring few exceptions, are hardly known to be popular, but Robert Kanigel has trodden that seemingly impossible path some three decades earlier to produce the most elegantly awe-inspiring tale of the last century, **The Man Who Knew Infinity**. His book, now converted into a Hollywood film, a psycho-analytic dual biography of Ramanujan and G.H. Hardy has, in its pursuit of capturing the quintessential Ramanujan through the odyssey of his romantic sojourn, has won many accolades since its publication and made the author a distinguished international celebrity. Kanigel's Magnum opus, is a biography that not only did justice to the phenomenal yet tragically short life of this genius, but also laid before the common reader an informed sketch of his stupendously huge Mathematical oeuvre that he had left as a legacy for the generations to come. No wonder, when Government of India declared 2012 as the National Mathematics Year to commemorate Ramanujan's 125th birth anniversary, the organizing committee formed for a yearlong celebration, along with many other mathematical activities throughout India, also took up the project of getting Kanigel's book translated into ten regional Indian languages under the auspices of Ramanujan Mathematical Society (RMS). The first one to come out was in Bengali (named *Ontoheener Antarzami*). It was published jointly by RMS and National Book Trust, India, and was formally inaugurated on 19th July 2013 at Kolkata, by the Nobel Laureate economist, Bharatratna Professor Amartya Sen, who was once a Master of Trinity College, of which

Ramanujan was the first Indian Fellow. Within a month, the Malayalam translation was released by the Chief Minister of Kerala. The Tamil and the Kannada translations were published on 22nd December 2013, the auspicious birthday of Ramanujan. Translations in Odia, Marathi and Gujrati followed gradually, but three other translations that were initiated, including the one in Hindi, are yet to be published. However, with the passage of time, the information related to Ramanujan, as one may find in Kanigel's well-researched book, comprehensive and up-to-date as it was during its time of publication in 1991, appeared to require further updating here and there. This is why, while working on the Bengali translation, it became almost a moral compulsion to try one's level best and shed some light on the recent boom in the mathematical research arena influenced by the unbelievably rich legacy of Ramanujan's mathematics, as well as some other socio-academic developments related to his name and life story that have taken place since then. It is with this mission at the back of mind, an addendum was proposed to this biography of epic proportions, which was duly accepted and appreciated by Robert Kanigel himself and finally got appended to the Bengali translation. The original article in English was later published in Mathematics Newsletter of Ramanujan Mathematical Society, in December 2013, (vol.24, No.3, pages 63-77)[*], while its revised and updated version was recently published in the mathematics magazine, *Bhāvanā* (Vol. 4, Issue 3, July 2020, pages 5-21)[https://bhavana.org.in/lasting-legacy-of-ramanujan-transcending-kanigels-canvas/]. The present article highlights only one or two issues from them, in a nutshell manner.

There have been many notable books on Ramanujan written in English after Kanigel, like the one by K. Srinivasa Rao (1998) or by S. Ram (2000). Another one, entitled 'The Mathematical Legacy of Srinivasa Ramanujan' was published in October, 2012. Written by eminent Ramanujan scholars, M. Ram Murty and V. Kumar Murty, this book has several chapters devoted to Ramanujan's conjecture and its impact on 20th and 21st century mathematics. In any such book on Ramanujan, mention of Kanigel in the bibliography is a common feature. On the other hand, Kanigel's passionate

[*] An interested reader may find a rather long list of some relevant papers in the Reference section of this article.

research is evident from the similar mentions of almost all the major Ramanujan related books published beforehand. However, there is a notable exception both ways, which seems worth mentioning. Almost at the same time of Kanigel's book, Wazir Hasan Abdi, a distinguished professor of Mathematics and a Fellow of The Indian National Science Academy has written a beautiful book on Ramanujan's life and work entitled '**Toils and Triumphs of Srinivasa Ramanujan—The Man and the Mathematician**'. Apart from containing almost all the information on Ramanujan's life that one may get in Kanigel's, it includes all the contribution that Ramanujan made to the Journal of Indian Mathematical Society before his voyage to Cambridge. Furthermore, in the last section there is a collection of six survey articles written by experts on various areas of Ramanujan's mathematical work. Subsequently, Bruce C. Berndt and Robert A. Rankin have published two wonderful books. The first one, called 'Ramanujan - Letters and Commentary', published in 1995 by AMS (Indian edition by Affiliated East West Press Private Limited, 1997) collects various letters written to, from, and about Ramanujan, and makes detailed commentaries on the letters. The second book, called 'Ramanujan - Essays and Surveys', published in 2001 by AMS-LMS (Indian edition by Hindusthan Book Agency, 2003) is a collection of excellent articles by various experts on Ramanujan's life and work. While writing a review of the Indian edition of the second book in volume 87 of Current Science in 2004, Professor M.S. Raghunathan remarked that "*our curiosity about the great is by no means confined to their lofty pursuits and achievements (perhaps because it is their common place experiences that will reassure us of our kinship with them). And in the case of Srinivasa Ramanujan, that curiosity is greater as the bare outlines of his life have so much of the romantic element. The excellent biography of Ramanujan by Robert Kanigel.... is indeed a comprehensive work; yet there remain many unanswered questions.*" Highlighting some such issues he further noted, "*A note by Berndt tells us that the original notebooks are to be found in the Librarian's office of the University of Madras..... We owe the publications of The Notebooks to the initiative of K. Chandrasekharan, one of our leading mathematicians – a fact that unfortunately finds no mention in much of the extensive writings about Ramanujan.*" Part three of this book has a short biography of Janakiammal, Ramanujan's wife, who eventually passed away on April 13, 1994 at the age of 94. An interview given by her to Pritish Nandy has been reproduced in the book, which reveals some glimpses of her personality. Part two of the book contains information regarding Ramanujan's illness and treatment. An article by Rankin reproduced here from the Proceedings of the Indian Academy of Sciences and another by a physician, D. A. B. Young, who did some investigations on this subject for the Royal Society, have considerable information not to be found in Kanigel's book. Raghunathan pointed out that, "*among other things, it appears that the contemporary diagnosis of Ramanujan's illness was not satisfactory.*" In an article in the Asia Pacific Mathematics Newsletter (April 2012, Vol. 2, No. 2), K. Srinivasa Rao of The Institute of Mathematical Sciences, Chennai, also pointed out the same. "*Mainly due to the efforts of Professor Robert A Rankin, a renowned mathematician and Dr. D. A. B. Young, a medical doctor, it is now common knowledge, amongst the admirers of Ramanujan, that the cause of the death of Ramanujan was not the then dreaded TB, but hepatic amoebiasis, which was the cause of his illness twice in his younger days, in India. Since TB was diagnosed by (some) doctors in England and in India after his return, as a celebrity he got the best medical attention and the full-fledged backing of the University of Madras. Since the treatment was done (not for hepatic amoebiasis but) for TB, it led to his premature death.*"

In his expertly written article full of medical jargon written to the Royal Society, Dr. Young pointed out that "*unfortunately, no official medical records of Ramanujan's illness during his time in England have survived, so any attempt at a retrospective diagnosis must depend on information in letters and reminiscences.*" And on the basis of such information, he retraced the whole medical history of Ramanujan's life and arrived at his claim of mis-diagnosis of tuberculosis. In his ten page long analysis of the diverse possibilities that the symptoms suffered by Ramanujan might indicate, Young took into account and examined all the available facts about Ramanujan's illness and the contradicting opinions of the Doctors who have attended him and finally presented what he called "*a diagnosis by exclusion*". Referring to the different possible diseases that might have caused his dominant symptom of 'Intermittent pyrexia', and eliminating them one-by-one through their would-be-evident manifestation in a blood count he reached the crux of his theory that the "*disease is hepatic amoebiasis and the fact that it has been arrived at here by a process of elimination should not disguise the high probability that it could have been the cause of intermittent fever in someone of Ramanujan's background.*"

However, Professor (Dr.) D. N. Guha Mazumder, a senior Gastroenterologist of high repute, who is a member of the Task Force of Liver Disease, Indian Council of Medical Research, New Delhi and former Head of the Department of Gastroenterology, Institute of Post Graduate Medical Education and Research, Kolkata, holds contrary views. After going through the medical arguments by Dr. Young in details, he referred to the 'diagnosis by exclusion' as proposed by Dr. Young as "far-fetched". He points out the fact that, at the time under consideration, Madras was quite advanced in tackling tropical diseases and with a blood count data suggesting any possibility of a liver abscess, it seems quite unlikely that the eminent doctors treating Ramanujan would have failed to explore that possibility as well. He further suggests that it is highly unlikely that a case of liver abscess, if not treated properly, would take such a prolonged period of time to worsen through gradual deterioration, rather than a faster manifestation of ultimate decline. He strongly feels that the available data are not sufficient to favour 'hepatic amoebiasis' against 'tuberculosis', for which Ramanujan was treated.

Another relatively recent interesting development relating to Ramanujan is noteworthy. It has been reported by most of his biographers including Kanigel, Abdi and Ranganathan, that during his Cambridge days, Ramanujan was very keen to communicate to his friend P. C. Mahalanobis, a glimpse of his own philosophical theory which involved an idea of multiplication between zero and infinity, the outcome of which would be the whole set of real numbers. From the reminiscence of Mahalanobis, as recorded by Ranganathan,

> Left to himself, he would often speak of certain philosophical questions. He was eager to work out a theory of reality which would be based on the fundamental concepts of "zero", "infinity" and the set of finite numbers. I used to follow in a general way

> but I never clearly understood what he had in mind. He sometimes spoke of "zero" as the symbol of the absolute (*Nirguna-Brahman*) of the extreme monistic school of Hindu philosophy, that is, the reality to which no qualities can be attributed, which cannot be defined or described by words, and which is completely beyond the reach of the human mind. According to Ramanujan, the appropriate symbol was the number "zero", which is the absolute negation of all attributes. He looked on the number "infinity" as the totality of all possibilities, which was capable of becoming manifest in reality and which was inexhaustible. According to Ramanujan, the product of infinity and zero would supply the whole set of finite numbers. Each act of creation, as far as I could understand, could be symbolized as a particular product of infinity and zero, and from each such product would emerge a particular individual of which the appropriate symbol was a particular finite number. I have put down what I remember of his views. I do not know the exact implication. He seemed to have been perhaps emotionally more interested in his philosophical ideas than in his mathematical work. He spoke with such enthusiasm about the philosophical questions that sometimes I felt he would have been better pleased to have succeeded in establishing his philosophical theories than in supplying rigorous proofs of his mathematical conjectures.

Mahalanobis could not fathom the mathematical idea behind Ramanujan's philosophical thinking, which seemed untenable in our usual set up of a number field, nor could any Ramanujan scholar since then interpret it, while some authors like Abdi, has even trodden an oversimplified path to profess that, perhaps Ramanujan had thought that any finite number divided by zero was infinity, whence he had concluded that zero multiplied by infinity would be any finite number. However, when in 2015, Avinash Sathaye, a professor of Mathematics at University of Kentucky came forward with a paper entitled "Bhāskarācārya's Treatment of the Concept of Infinity", published in *Gaṇit Bhāratī, Vol. 37, No. 1-2 (2015) pages 55-67*, it was immediate that, when Sathaye's interpretation of the concepts of *khahara* and *khaguna* as per Bhāskarācārya is seen in the light of 'idempotence' sitting at the heart of the new algebraic structure proposed by Sathaye, not only the sums of *Bījagaṇita* that were so long being considered to be "wrongly framed" by Bhāskarācārya, got commensurately interpreted in the new mathematical structure (which is neither a field nor a lattice), but also this new algebraic structure perfectly matches Ramanujan's apparently bizarre mathematical idea, that came to us through the reminiscence of

Mahalanobis.

To commemorate Ramanujan's death centenary this year, Vigyan Prasar, an autonomous body under the Department of Science and Technology, Govt. of India, led by its director Nakul Parashar, decided to take up what they called '**Ramanujan Yatra**', a large-scale mathematics awareness and popularization campaign for high school and college students, mathematics teachers, communicators and enthusiasts, covering every direction, nook and corner of India. Towards organizing the events in a pan India fashion, a National Core Committee, of which the present author is a member, was formed with T.V.Venkateswaran as its national coordinator. In its first meeting held on 11th February, 2020 at New Delhi, the committee chalked out an eight month long program of different series of lecture demonstrations by eminent scholars, math-science fair(s) and expo(s). These were planned to be held once in every month as a two/three day program in different cities across India, to be inaugurated on 26th April, 2020, the day of his untimely sad demise, with a befitting program at Kumbakonam, the place where Ramanujan was brought up and to culminate the 'Yatra' with a final program on coming 22nd December 2020, the National Mathematics Day, at New Delhi. But alas! Due to the outbreak of Covid-19 pandemic and subsequent nationwide lock-down, the initial program at Kumbakonam had to be organized locally in a very low key manner, maintaining social distance with a handful people, at the Town High School campus and also at Government Arts College campus, where Ramanujan studied. The core committee then reformulated their plan to e-mode towards holding Webinars on Life and Works of Ramanujan across the country. As this article is going to press, several such talks in a variety of Indian regional languages have already been organized and uploaded in Youtube, while famous Ramanujan Scholars like Bruce Berndt, Ken Ono, M. Ram Murty, Don Zagier, Sujatha Ramadorai, R. Balasubramanian have given their Webinars in English.[https://www.mtai.org.in/events/srinivasa-ramanujans-centenary-memorial-virtual-yatra-talk-series/]

Ramanujan left us exactly 100 years ago on 26th April 1920. But he still lives and will continue to live through his immortal work. Indeed, the impact of the story of his life as a fountainhead of inspiration on the society as a whole, seems to be growing day by day. The immense significance of his final work done almost from deathbed reverberates in many a comment by great scholars of our time. "*The mock-theta functions give us tantalizing hints of a grand synthesis still to be discovered. This remains a challenge for the future*", said Freeman Dyson of Institute of Advanced Study, Princeton, while Fields Medalist Manjul Bhargava of Princeton University pointed out that "*We're still trying to understand some of his work today — and when we do, it often transforms entire areas of mathematics*". Let us end by quoting Robert Kanigel: "*What Ramanujan did will live forever. It will not, to be sure, live in the hearts of the masses of men, like the work of Gandhi, Shakespeare or Bach. Still, his ideas and discoveries, percolating through those few mind tuned to them, will mingle with the intellectual energy of the cosmos, and thence into the deep, broad pool of human knowledge.*"

Ramanujans never die.

International Academic Programme Committee

- Alladi Krishnaswami, University of Florida, USA
- Bimal Roy, Indian Statistical Institute, India
- Carsten Thomassen, Technical University of Denmark, Denmark
- Dhruv Mubayi, University of Illinois, USA
- George Andrews, Pennsylvania State University,USA
- Gyula O. H. Katona, Hungarian Academy of Sciences, Alfréd Rényi Institute of Mathematics, Hungary
- Maria Chudnovsky, Princeton University, USA
- Michel Waldschmidt, Sorbonne Université,Institut Mathématique de Jussieu,FRANCE
- Pavol Hell, Simon Fraser University, Canada
- R Balasubramanian, National Centre for Mathematics, Indian Institute of Technology, Mumbai, India
- R.B. Bapat, Indian Statistical Institute, India
- Rekha R Thomas, University of Washington, USA
- Sandi Klavžar, University of Ljubljana, Slovenia
- Tetali Prasad, Georgia Institute of Technology, USA
- Arvind V, Institute of Mathematical Sciences,India
- Ambat Vijayakumar, Cochin University of Science & Technology, India - Chair

Advisory Committee

Patrons:

- Fr. Benny Nalkara CMI, Manager, Rajagiri School of Engineering & Technology, India
- Rev. Fr. Dr. Jose Kuriedath , Director, Rajagiri School of Engineering & Technology, India
- Rev.Fr. John Therezhath CMI, Administrator, Rajagiri School of Engineering & Technology, India
- S. Ponnusamy, IIT Madras, President Ramanujan Mathematical Society, India

Members:

- C.S. Aravinda, TIFRCAM, Bangalore, Secretary, Ramanujan Mathematical Society, India
- E. Sampathkumar, Mysore University, India
- K. Poulose Jacob, Dean Research, Rajagiri School of Engineering & Technology, India
- Kuldeep Singh Charak, Jammu University, Academic Secretary, Ramanujan Mathematical Society, India
- P.S. Sreejith, Principal, Rajagiri School of Engineering & Technology, India
- R. Balakrishnan, Bharathidasan University, India
- R. Sivaramakrishnan, Retd. Professor, University of Calicut, India
- Saber Elaydi, Trinity University, USA
- T.Thrivikraman, Retd. Professor, Cochin University of Science & Technology, India
- V. Kannan, S R M University, India

Organising Committee

- Abraham Thomas, RSET
- Ajith B., RSET
- Anto P. O., RSET
- Benny V. J., RSET
- Biju Paul, RSET
- Bindhu Paul, RSET
- Bindu V. A., RSET
- Binu A., RSET
- Binu R., RSET
- Bobby Anto, RSET
- Cyriac Thomas, RSET
- Denny Thomas, RSET
- Dhanya P. M., RSET
- Elizabeth Rita Samuel, RSET
- Eva Cherian, RSET
- Fr. Jaison Paul CMI, RSET
- Fr. Joel George CMI, RSET
- Fr. Joseph C. CMI, RSET
- Fr. Mejo Paul CMI, RSET
- Fr. Thomas P. J. CMI, RSET
- Fr. Varghese CMI, RSET
- Hari C. V., RSET
- Jaya Abraham, RSET
- Jobin Jose, RSET
- Jomon P. J., RSET
- Jomy K. J., RSET
- Joseph P. M., RSET
- Krishnadas N., RSET
- Kuriakose M. T., RSET
- Lindamol Joseph, RSET
- Manoj Tharian, RSET
- Maria Poulose, RSET
- Mary Reshmi, RSET
- Neeba E. A., RSET
- Nitheesh Kurian, RSET
- Paul M Jacob, RSET
- Radhika Das, RSET
- Ramkumar P. B., RSET
- Rejeesh T. Chacko, RSET
- Reji V. V. , RSET
- Reya Kuruvila, RSET
- Rijo K. Varghese, RSET
- Rithu James, RSET

- Sabeesh Robinson, RSET
- Saji Joseph, RSET
- Sheebu Jacob, RSET
- Shiju Kuriakose, RSET
- Shiju M Nair, RSET
- Shyam Sunder Iyer, RSET
- Sonia Paul, RSET
- Thomas T. J., RSET
- Tomy Thomas, RSET
- Tony Joseph, RSET
- Varun Jose, RSET
- Vincent K. John, RSET
- Vinmol Dominic, RSET
- Vinod Kumar P. B. (RSET - Coordinator)

Schedule

IST		**1000- 1045**	**1115 - 1145**	**1215-1300**	**1430 - 15.00**	**1515-1645**	**1700 -1745**	**1800-1845**	**19.00 – 19.45**	**20.00-21.00**
Day 1		P1	IT1 MM	P2	IT3 OA	CS	P3	P4	P5	PL
DEC 11		BR	IT2 KC	VA	IT4 BS		DM	PS	GA	MR
Time at speaker's place		IST	0815 & IST	IST	1100 & IST		0730	0830	0930	
IST	**0930 - 1015**	**1045-1115**	**1145-1215**	**1245-1315**	**1415-1645**	**1715-1800**	**1830-1915**	**1945-2030**		
Day 2	P6	IT5 YS	IT7 SW	IT9 MS	CS	P7	P8	P9		
DEC 12	RT	IT6 MK	IT8 SG	IT10 VR		PC	VK	DW		
Time at speaker's place		1315 & IST	0815 & IST	0915 & IST		2345	0900	1015		
IST	**0930 - 1015**	**1045-1130**	**1200-1245**	**1345-1430**	**1445-1700**	**1730-1815**	**1845-1915**	**1945-2030**		
Day 3	P10	P11	P12	P13	CS	P14	IT11 SC	P15		
DEC 13	AK	NS	KM	YN		NK	IT12 FT	KS		
Time at speaker's place	1200	IST	1530	1015		0800	0915	1015		
IST	**0930-1015**	**1045-1130**	**1200-1245**	**1345-1430**	**1500-1545**	**1615-1700**	**1730-1815**	**1845-1930**	**1930-2000**	
Day 4	P16	P17	P18	P19	P20	P21	P22	P23	Closing	
DEC 14	KK	RM	MW	GK	SK	JP	BM	TP		
Time at speaker's place	1300	IST	0830	0930	1130	1245	0800	0915		

CS: CONTRIBUTED SESSION PL: PUBLIC LECTURE

PUBLIC LECTURE: RAMANUJAN, NUMBER THEORY AND PANDEMIC

SPEAKER :M. RAM MURTY (MR) **CHAIR** : C. S. ARAVINDA

*PLENARY SESSIONS – 45 MINUTES, INVITED SESSIONS- 30 MINUTES

PLATFORM: ZOOM

INAUGURAL CEREMONY : 10 DECEMBER 2020, 19 30 - 20 30

DAY 1, 11 DECEMBER 2020

SESSION	**SPEAKERS**	**CHAIR**	**TIME**
P1	R. BALASUBRAMANIAN	RITABRATA MUNSHI	10 00 - 10 45
IT1	MARTIN MILANIC	TAMIZH CHELVAM	11 15 - 11 45
IT2	KALYAN CHAKRABORTY	B.SURY	11 15 - 11 45
P2	V. ARVIND	NITIN SAXENA	12 15 - 13 00
IT3	OSWIN AICHHOLZER	MANOJ CHANGAT	14 30 - 15 00
IT4	B. SURY	KALYAN CHAKRABORTY	14 30 - 15 00
	CONTRIBUTED SESSION		15 15 - 16 45
P3	DHRUV MUBAYI	V. ARVIND	17 00 - 17 45
P4	PETER SARNAK	R. BALASUBRAMANIAN	18 00 - 18 45
P5	GEORGE ANDREWS	V. ARVIND	19 00 - 19 45
PL	M. RAM MURTY	C. S. ARAVINDA	20 00 - 21 00

DAY 2, 12 DECEMBER 2020

SESSION	**SPEAKERS**	**CHAIRS**	**TIME**
P6	REKHA THOMAS	SANOLI GUN	09 30 - 10 15
IT5	YOUNGTANG SHI	TAMIZHCHELVAM	10 45 - 11 15
IT6	MAHESH KAKDE	SANOLI GUN	10 45 - 11 15
IT7	STEPHEN WAGNER	H. S. RAMANE	11 45 - 12 15
IT8	SANOLI GUN	KALYAN CHAKRABORTY	11 45 - 12 15
IT9	MATHIAS SCHACHT	P. B. VINODKUMAR	12 45 - 13 15
IT10	VENKATESH RAMAN	MANOJ CHANGAT	12 45 - 13 15
	CONTRIBUTED SESSION		14 15 - 16 45
P7	PETER CAMERON	R.BALAKRISHNAN	17 15 - 18 00
P8	V. KUMAR MURTY	MAHESH KAKDE	18 30 - 19 15
P9	DOUGLAS B. WEST	S. ARUMUGAM	19 45 - 20 30

DAY 3, 13 DECEMBER 2020

SESSION	SPEAKERS	CHAIRS	TIME IST
P10	ALLADI KRISHNASWAMI	R. BALASUBRAMANIAN	09 30 - 10 15
P11	NITIN SAXENA	V. ARVIND	10 45 - 11 30
P12	KOHJI MATSUMOTO	B. SURY	12 00 - 12 45
P13	YURI NESTERENKO	B. SURY	13 45 - 14 30
	CONTRIBUTED SESSION		14 45 - 17 00
P14	NEAL KOBLITZ	NITIN SAXENA	17 30 - 18 15
IT11	SEBASTIAN CIOABA	INDULAL. G	18 45 - 19 15
IT12	FRANK THORNE	MAHESH KAKDE	18 45 - 19 15
P15	KANNAN SOUNDARARAJAN	R. BALASUBRAMANIAN	19 45 - 20 30

DAY 4, 14 DECEMBER 2020

SESSION	SPEAKERS	CHAIRS	TIME IST
P16	KEN-ICHI KAWARABAYASHI	R. B. BAPAT	09 30 - 10 15
P17	RITABRATA MUNSHI	KALYAN CHAKRABORTY	10 45 - 11 30
P18	MICHEL WALDSCHMIDT	SANOLI GUN	12 00 - 12 45
P19	G. O. H. KATONA	S. ARUMUGAM	13 45 - 14 30
P20	SANDI KLAVZAR	MANOJ CHANGAT	15 00 - 15 45
P21	JANOS PACH	R. B. BAPAT	16 15 - 17 00
P22	BOJAN MOHAR	R. B.BAPAT	17 30 - 18 15
P23	TETALI PRASAD	R. BALAKRISHNAN	18 45 - 19 30
	CLOSING		19 30 - 20 00

SCHEDULE OF CONTRIBUTED SESSION – 15 MIN EACH

	TRACK	**CHAIR**	**ABSTRACT NUMBERS**
DAY 1 11 DEC 2020 15 15 - 16 45	I	ANUP. B. DIXIT CMI, CHENNAI	12, 24, 41, 59, 86, 88
DAY 2 12 DEC 2020 14 15 - 16 45	I	H. S. RAMANE KARNATAK UNIVERSITY DHARWAD	38, 45, 08, 13, 20, 22, 46, 48, 61
	II	TAMIZH CHELVAM M. S. UNIVERSITY TIRUNELVELI	30, 43, 52, 56, 14, 60, 04
DAY 3 3 DEC 2020 14 45 - 17 00	I	MANOJ CHANGAT UNIVERSITY OF KERALA TRIVANDRUM	17, 31, 44, 50, 53, 54, 57, 64
	II	INDULAL. G ST.ALOYSIUS COLLEGE EDATHUA	18, 40, 58, 67, 69, 70, 71, 75, 83

Plenary Talks

Ramanujan's legacy: The work of the SASTRA prize winners

Krishnaswami Alladi

University of Florida, USA

The SASTRA Ramanujan Prize, launched in 2005, is a $10,000 annual award given to mathematicians not exceeding the age of 32, for path-breaking contributions in areas in uenced by Srinivasa Ramanujan. The age limit has been set at 32 because Ramanujan lived only for 32 years and in that brief life span made revolutionary contributions; so the challenge for the prize candidates is to show what they have achieved in that same time frame! The prize is given each year at SASTRA University in Kumbakonam (Ramanujan's hometown) in South India around December 22 (Ramanujan's birthday). The prize has been unusually effective in recognizing extremely gifted mathematicians at an early stage of their careers, and so is now considered to be one of the most prestigious and coveted mathematics awards in the world. The list of winners and their affiliation at the time of receiving the award is:

2005: Manjul Bhargava (Princeton) and Kannan Soundararajan (Michigan) - two full prizes, not shared

2006: Terence Tao (UCLA)

2007: Ben Green (Cambridge)

2008: Akshay Venkatesh (Stanford)

2009: Kathrin Bringmann (Cologne)

2010: Wei Zhang (Harvard)

2011: Roman Holowinsky (Ohio State)

2012: Zhiwei Yun (MIT and Stanford)

2013: Peter Scholze (Bonn)

2014: James Maynard (Oxford and Montreal)

2015: Jacob Tsimerman (Toronto)

2016: Kaisa Matomaki Turku, Finland) and Maksym Radziwill (McGill and Rutgers) - shared

2017: Maryna Viazovska (Ecole Polytechnique, Lausanne)

2018: Yifeng Liu (Yale) and Jack Thorne (Cambridge) - shared

2019: Adam Harper (Warwick)

2020: Shai Evra (Princeton and Hebrew University)

I will describe briefly the spectacular work for which the awardees were recognised and focus on some aspects of their research that relate to Ramanujan.

Keywords: SASTRA Ramanujan Prize, Work of the Winners, Ramanujan's Legacy

Univariate ideal membership and the combinatorial nullstellensatz

V. Arvind

The Institute of Mathematical Sciences (HBNI), India

In this talk we will discuss the computational complexity of testing membership in univariate ideals: Let $R = \mathbb{F}[x_1, x_2, \ldots, x_n]$ be the polynomial ring over the field $\mathbb{F}$ in variables $x_1, x_2, \ldots, x_n$. Let $I \subset R$ be the ideal generated by univariate polynomials $p_1(x_1), p_2(x_2), \ldots, p_n(x_n)$. Given a polynomial $f(x_1, x_2, \ldots, x_n) \in R$ by an arithmetic circuit, the algorithmic problem is to test if $f \in I$. We will motivate the problem with examples, bringing out connections to Alon's Combinatorial Nullstellensatz, and we will describe the following results:

- Over the field of rationals the problem is hard for the first level of the Counting hierarchy. Over finite fields of characteristic k, the problem is hard for the modular counting hiearchy $\mathrm{Mod}_k\mathrm{P}$, and is in fact located in randomized version of $\mathrm{Mod}_k\mathrm{P}$.

- If f is a degree-d, rank-r polynomial it has a randomized $d^{O(r)} \cdot \mathrm{poly}(n)$ time algorithm. This yields a new $n^{O(r)}$ algorithm for evaluating the permanent of a $n \times n$ matrix of rank r, over any field $\mathbb{F}$.

- Over rationals, when the ideal $I = \langle x_1^{e_1}, \ldots, x_n^{e_n} \rangle$, the problem can be solved in randomized $O^*((2e)^d)$ time. On the other hand, if each p_i has distinct rational roots we can solve it in randomized $O^*(n^{d/2})$ time.

- Finally, if $I = \langle p_1(x_1), \ldots, p_k(x_k) \rangle$, with k as fixed parameter, then the problem is hard for the second level of the W-hierarchy. The problem is MINI[1]-hard in the special case when $I = \langle x_1^{e_1}, \ldots, x_k^{e_k} \rangle$.

This is joint work with Abhranil Chatterjee (IMSc), Rajit Datta (CMI), and Partha Mukhopadhyay (CMI).

Keywords: Ideal membership, Algorithms, Combinatorial Nullstellensatz

Geodesic drawings of graphs and their limits

Bojan Mohar

Simon Fraser University, Canada

The study of crossing numbers of graphs has gone a long way [3], but some of the very basic questions are still far from being understood. There is an increasing interest to consider drawings of graphs as geometric or combinatorial objects on their own. The speaker will introduce the notion of geodesic drawings and their limits in parallel to the graph limits theory of Lovász et al. [2]. Basic properties will be discussed.

Keywords: Graph drawing, Crossing number, Graph limit, Geodesic drawing

[1] Marthe Bonamy, Bojan Mohar, and Alexandra Wesolek, Limiting crossing numbers for geodesic drawings on the sphere, GD 2020. arXiv: 2008.10459

[2] László Lovász, Large Networks and Graph Limits, American Mathematical Society, Colloquium Publications, Volume 60, 2012.

[3] Marcus Schaefer, Crossing Numbers of Graphs, CRC-Press, 2018.

Recent advances in Ramsey theory

Dhruv Mubayi

University of Illinois, USA

Ramsey theory studies the paradigm that every sufficiently large system contains a well-structured subsystem. Within graph theory, this translates to the following statement: for every positive integer s, there exists a positive integer n so that for every partition of the edges of the complete graph on n vertices into two classes, one of the classes must contain a complete subgraph on s vertices. Beginning with the foundational work of Ramsey in 1928, the main question in the area is to determine the smallest n that satisfies this property.

For many decades, randomness has proved to be the central idea used to address this question. Very recently, we proved a theorem which suggests that "pseudo-randomness" and not complete randomness may in fact be a more important concept in this area. This new connection opens the possibility to use tools from algebra, geometry, and number theory to address the most fundamental questions in Ramsey theory. This is joint work with Jacques Verstraete.

Reconstruction of graphs from the deck of k-vertex induced subgraphs

Douglas B. West

University of Illinois, USA

The k-deck of a graph G is its multiset of subgraphs induced by k vertices. Letting $n = |V(G)|$, the famous Reconstruction Conjecture of Kelly [6] and Ulam [20] from 1942 is that the $(n-1)$-deck determines G when $n \geq 3$. Survey papers concerning this classical problem include [2,12,13].

The notion we study here is based on the following.

Observation 1. The k-deck of any graph determines its $(k-1)$-deck.

Thus an enhanced version of the reconstruction problem is to seek the least k such that the k-deck determines G. A graph is *ℓ-reconstructible* if it is determined by its $(n-\ell)$-deck, meaning that no graph not isomorphic to it has the same $(n-\ell)$-deck. Kelly [7] extended the Reconstruction Conjecture:

Conjecture 2. [[7]] For $\ell \in \mathbb{N}$, there is a threshold M_ℓ such that all graphs with at least M_ℓ vertices are ℓ-reconstructible.

The classical Reconstruction Construction is $M_1 = 3$. Nýdl [17] proved that M_ℓ, if well-defined, must grow at least superlinearly in ℓ.

We survey past results in this topic along with more recent results that are joint with H. Spinoza [18] and with A. Kostochka, M. Nahvi, and D. Zirlin [8–10]. For $\ell \leq (1-o(1))n/2$, almost every graph is ℓ-reconstructible [18], generalizing the old result of [1,4,15] for $\ell = 1$. However, when $\ell = n/2$, the $(n-\ell)$-deck may not even determine whether G is connected. In particular, the path with 2ℓ vertices has the same ℓ-deck as the disjoint union of a path with $\ell-1$ vertices and a cycle with $\ell+1$ vertices.

We can also study ℓ-reconstructibility of graph properties or families. A property is ℓ-reconstructible if the $(n-\ell)$-deck of an n-vertex graph determines whether the graph satisfies that property. Taylor [19] proved that the degree list is ℓ-reconstructible when $n \geq (1+o(1))e\ell$. Spinoza and West [18] proved that the property of connectedness is ℓ-reconstructible when $n \geq 2\ell^{(\ell+1)^2}$, but this is certainly not sharp. Connectedness [14] and the degree list [3] are 2-reconstructible when $n \geq 6$. Here the answer is also known for $\ell = 3$:

Theorem 3. [[8]] The connectedness and the degree list of an n-vertex graph are 3-reconstructible when $n \geq 7$, and these thresholds are sharp.

More is known about special families. Graphs with n vertices in which every component has at most $n - \ell$ vertices are ℓ-reconstructible, but for graphs having a component with $n - \ell + 1$ vertices, ℓ-reconstructibility is equivalent to the original Reconstruction Conjecture [11]. Every complete r-partite graph is reconstructible from its $(r+1)$-deck [18]. Every 3-regular graph is 2-reconstructible [9], and r-regular graphs that are not 2-connected are $(r + 1)$-reconstructible [11].

Essentially everything is known for graphs with maximum degree 2. Spinoza and West [18] proved that every n-vertex graph with maximum degree 2 is $\lfloor (n - 1)/2 \rfloor$-reconstructible, and this is sharp. Furthermore, for every graph with maximum degree 2, the least k such that G is reconstructible from its k-deck is known: it is approximately $\max\{m/2, m'\}$, where m and m' are the sizes of the largest and next largest components. The main tool for the lower bounds is this:

Theorem 4. [[18]] Let G and G' be graphs with maximum degree 2 having the same number of vertices and the same number of edges. If every component in each graph is a cycle with at least $k + 1$ vertices or a path with at least $k - 1$ vertices, then G and G' have the same k-deck.

Reconstructibility of trees is of particular interest. Nýdl [16] provided nonisomorphic trees with 2ℓ vertices having the same ℓ-deck, thereby showing that the threshold for ℓ-reconstructibility of trees is at least $2\ell + 1$ (for $\ell \geq 3$). He conjectured that when $n \geq 2\ell + 1$, no two n-vertex trees have the same $(n - \ell)$-deck (for ℓ-reconstructibility, one must also show that all reconstructions are trees). Giles [5] proved that trees with at least six vertices are 2-reconstructible. With a very long argument, we have proved.

Theorem 5. [[10]] Trees with at least 22 vertices are 3-reconstructible.

The sharp threshold should be $n \geq 7$. Finally, for general ℓ, Zirlin [21] has proved that when n is at least about 2.5ℓ, the property of being a tree is ℓ-reconstructible.

References

[1] B. Bollobás, Almost every graph has reconstruction number three, *J. Graph Theory* 14 (1990), 1–4.

[2] J. A. Bondy, A graph reconstructor's manual, in *Surveys in Combinatorics (Guildford, 1991)*, Lond. Math. Soc. Lec. Notes 166 (Cambridge U. Press, 1991), 221–252.

[3] Zh. A. Chernyak, Some additions to an article by B. Manvel: "Some basic observations on Kelly's conjecture for graphs" (Russian), *Vestsī Akad. Navuk BSSR Ser. Fīz.-Mat. Navuk* (1982), 44–49, 126.

[4] P. Chinn, A graph with p points and enough distinct $(p-2)$-order subgraphs is reconstructible, *Recent Trends in Graph Theory* Lecture Notes in Mathematics 186 (Springer, 1971).

[5] W. B. Giles, Reconstructing trees from two-point deleted subtrees, *Discrete Math.* 15 (1976), 325–332.

[6] P. J. Kelly, On isometric transformations, PhD Thesis, University of Wisconsin-Madison, 1942.

[7] P. J. Kelly, A congruence theorem for trees, *Pacific J. Math.* 7 (1957), 961–968.

[8] A. V. Kostochka, M. Nahvi, D. B. West, and D. Zirlin, Degree lists and connectedness are 3-reconstructible for graphs with at least seven vertices, *Graphs and Combinatorics* 36 (2020), 491-501.

[9] A. V. Kostochka, M. Nahvi, D. B. West, and D. Zirlin, 3-Regular graphs are 2-reconstructible, *European J. Combinatorics* 91 (2021) 103216 (no page numbers), (published on-line September, 2020, DOI: 10.1016/j.ejc.2020.103216).

[10] A. V. Kostochka, M. Nahvi, D. B. West, and D. Zirlin, Trees and rooted trees are 3-Reconstructible, in preparation.

[11] A. V. Kostochka and D. B. West, On reconstruction of graphs from the multiset of subgraphs obtained by deleting ℓ vertices, *IEEE Transactions on Information Theory* (published on-line March, 2020, DOI: 10.1109/TIT.2020.2983678).

[12] J. Lauri, Pseudosimilarity in graphs—a survey, *Ars Combin.* 46 (1997), 77–95.

[13] J. Lauri and R. Scapellato, *Topics in Graph Automorphism and Reconstruction*, *London Math. Soc. Student Texts 54*, (Cambridge Univ. Press, Cambridge, 2003). Second edition: *London Math. Soc. Lect. Note Series 432*, (Cambridge Univ. Press, Cambridge, 2016).

[14] B. Manvel, Some basic observations on Kelly's conjecture for graphs, *Discrete Math.* 8 (1974), 181–185.

[15] V. Müller, Probabilistic reconstruction from subgraphs, *Comment. Math. Univ. Carolinae* 17 (1976), 709–719.

[16] V. Nýdl, A note on reconstructing finite tree from small subtrees, *Acta Univ. Carol. Math. Phys.* 31 (1990), 71–74.

[17] V. Nýdl, Finite undirected graphs which are not reconstructible from their large cardinality subgraphs, *Discrete Math.* 108 (1992), 373–377.

[18] H. Spinoza and D. B. West, Reconstruction from the deck of k-vertex induced subgraphs, *J. Graph Theory* 90 (2019), 497–522.

[19] R. Taylor, Reconstructing degree sequences from k-vertex-deleted subgraphs. *Discrete Math.* 79 (1990), 207–213.

[20] S. M. Ulam, A collection of mathematical problems, *Interscience Tracts in Pure and Applied Mathematics* 8 (Interscience Publishers, 1960).

[21] D. Zirlin, in preparation.

How Ramanujan may have thought of the mock theta functions?

George E. Andrews

The Pennsylvania State University, USA

The mock theta functions made their first appearance in Ramanujan's last letter to Hardy. Ramanujan explains that he is trying to find functions apart from theta functions that behave like theta functions near the unit circle. Where did he ever get the idea that such functions might exist? Why in the world would he consider the special q-series that he lists in his last letter? The object of this talk is to provide a plausible explanation for the discovery of mock theta functions.

Keywords: mock theta functions, Ramanujan, theta functions

Reference: G. E. Andrews, How Ramanujan May have Discovered the Mock Theta Functions , Philosophical Transactions for the Royal Society A(2019) 378:20180436

A study in Scarlet and other shades of red

Gyula O.H. Katona

Rényi Institute, Hungarian Academy of Sciences, Hungary

Suppose that in a certain n-element population (denoted by $[n]$ there is exactly one infected person. Obviously, one has to take a blood or saliva sample from every individual, therefore the expenses can be reduced only reducing the number of chemical tests. It was observed by Dorfman [2] and Sterrett [10] that it is not necessary to test every sample one by one, but one can form subsets of the samples, testing them together. This method is called "group testing" or "combinatorial search".

Mathematically the model is the following. Subsets $A \subset [n]$ can be tested if the unknown element x is in A or not. The unknown x is to be found on the basis of the answers. There are two main versions. When the next test set is chosen depending the previous answers, the search is called *adaptive*, while in the case when the family $A_1, A_2, \ldots, A_m$ of test sets is given in advance then the search is called *non-adaptive*. It is easy to see that, if any subset $A \subset [n]$ can be used as a test set then the search can be carried out in $\lceil \log n \rceil$ steps (even in the non-adaptive case). However in practical situation the test sets can be chosen only from a family $\mathcal{A}$ of subsets of $[n]$. Rényi suggested to find the minimum number of tests if $\mathcal{A}$ consist of all sets of size at most k when k is a relatively small integer. (This is a natural assumption in the case of finding the infected person.) Both the adaptive and the non-adaptive cases were solved in [4]. The non-adaptive case can be formulated in the following way: find the minimum number of subsets (of $[n]$ of at most k elements such that for any two distinct elements $x, y \in [n]$ there is a set separating them.

Let us note that there are many practical situations when this model is applicable. For instance when a failing part has to be found in a complicated device (e.g the human body of the sick person), or in a criminal investigation when the perpetrator should be found from the set of possible suspects. An old survey [5] and a newer book [3] shows many applications and variants of the theoretical model above.

One of the important variants is when the answer to the question "is $x \in A$?"

can be incorrect. Because of a human error or some dirt spoils the chemical test. This is called *search in presence of a liar* or the *Rényi-Ulam game*. Here, again, one is looking for the unknown element $x \in [n]$ by asking questions of form "is $x \in A$?". However it is supposed that at most ℓ answers can be incorrect. Yet, the unknown element should be found without an error. The mathematical problem here is to find the minimum number of tests needed (in both, adaptive and non-adaptive cases). A good survey was written by Deppe [1], some open problems can be found in [6].

In the cases above, in the traditional model of "search in presence of liar", the lies come independently. In some applications, however there might be connections among the test, the unknown x and the lie. For instance when x is the perpetrator in a criminal investigation then the answer to a question to the eyewitness can be incorrect depending on x. Say, if x is a friend of the witness. This situation can be described in the following way. Here instead of asking "is $x \in A$ or in its complement?" there is a certain subset L and the answer can be incorrect if x is in L. Otherwise it is surely correct. Mathematically in this case the following model is considered. There is exactly one unknown element in $[n]$. A question is a partition of $[n]$ into three classes: (A, L, B). If $x \in A$ then the answer is "yes" (or 1), if $x \in B$ then the answer is "no" (or 0), finally if $x \in L$ then the answer can be either "yes" or "no". In other words, if the answer "yes" is obtained then we know that $x \in A \cup L$ while in the case of "no" answer the conclusion is $x \in B \cup L$. The mathematical problem is to minimize the minimum number of questions under certain assumptions on the sizes of A, B and L. This problem has been solved under the condition $|L| \geq r$ by the author and Krisztián Tichler in previous papers [7], [8] for both the adaptive and non-adaptive cases.

In the present work we suggest to solve the problems under the conditions $|A| \leq a, |B| \leq b$. The adaptive case is completely solved. Let us make clear that the problem in the non-adaptive case is a problem of Extremal Set Theory. The minimum number of partitions (A, L, B) should be determined under the conditions $|A| \leq a, |B| \leq b$ and that for any two distinct elements $x, y \in [n]$ there is a partition strongly separating them that is one of the is in A, the other one is in B. We present asymptotic solutions. Among others the concept of *graph entropy* is used what was introduced by Körner (see [9]).

Keywords: combinatorial search, extremal set theory

References

[1] Deppe, c., Coding with feedback and searching with lies, in *Entropy, Search, Complexity*, 2007, Springer.

[2] Dorfman, R., The detection of defective members of large populations, *Ann. Math. Stat.* **14**(1943) 436-440.

[3] Du, D., Hwang, FK. and Hwang, F., *Combinatorial group testing and its applications*, World Scientific, 2000.

[4] Katona, K., On separating systems of a finite set, *J. Combin. Theory* **1**(1966) 174-194.

[5] Katona, G.O.H., Combinatorial Search Problems, in *A survey of combinatorial theory* (Ed. by J.N. Srivastava, North Holland/American Elsevier, Amsterdam/New York, 1973) 285-308.

[6] Katona, G.O.H., Search with small sets in presence of a liar, *J. Statictical Planning and Inference* **100**(2002) 319-336.

[7] Katona, G.O.H. and Tichler, K., Search when the lie depends on the target, in: *Information Theory, Combinatorics, and Search Theory* (In Memory of Rudolf Ahlswede) (H. Aydinian, F. Cicalese, Ch. Deppe, Eds.) LNCS 7777, Springer, 2013, 524-530.

[8] Katona, G.O.H. and Tichler, K., Existence of a maximum balanced matching in the hypercube, *J. Statist. Theory and Practice* **7**(2013) 617-62.

[9] Simonyi, G., Graph entropy: a survey, in *Combinatorial Optimization*, : Papers from the DIMACS Special Year (eds. William Cook, László Lovász, Paul D. Seymour), 1995.

[10] Sterrett, A., On the detection of defective members of large populations, *Ann. Math. Stat.* **28**(1957) 1033.

Low-dimensional combinatorics

János Pach

Rényi Institute, Hungary

Several fundamental results in extremal graph and hypergraph theory were motivated by their prospective geometric applications. For example, the Erdős-Szekeres Happy Ending Problem about convex polygons has inspired Ramsey theory, and, in a roundabout way, also led to the discovery of Turán's theorem. Erdős's famous number-theoretic questions and his celebrated conjectures about the distribution of distances among n points have served as a driving force behind a lot of research in structural graph theory, culminating in Szemerédi's regularity theory. Although many of these theories and techniques were developed to deal with geometric questions, their direct applications to the original problems rarely yield optimal results.

What is behind this paradoxical phenomenon? Our results during the past ten years suggest that the combinatorial structures (graphs and hypergraphs) associated with the underlying geometric arrangements are frequently in some well-defined sense: They are (1) semi-algebraic with bounded complexity; they have bounded (2) Vapnik-Chervonenkis dimension; or they can be described using a bounded number of linear orders, that is, the (3) Dushnik-Miller dimension of the associated partially ordered set is bounded by a constant.

In this talk, we illustrate by a number of recent examples, how under these assumptions one can strengthen many important combinatorial results and approach some notoriously difficult open problems such as the Schur-Erdős problem, the Erdős-Hajnal conjecture, or the sunflower conjecture.

Keywords: Vapnik-Chervonenkis dimension, semialgebraic graph, partially ordered set, Erdős-Hajnal conjecture

Almost all entries in the character table of the symmetric group are divisible by any given prime

Kannan Soundararajan

Stanford University, USA

Based on numerical evidence, A. Miller conjectured recently that almost all entries in the character table of the symmetric group are divisible by any given prime number. I will report on work with Sarah Peluse which establishes this conjecture. The proof relies upon an understanding of the structure of typical partitions a large number N, a subject originating in the celebrated work of Hardy and Ramanujan obtaining an asymptotic formula for $p(N)$.

Keywords: character table, symmetric group, divisibility, partitions

Directed graph minor

Ken-ichi Kawarabayashi

National Institute of Informatics, Tokyo, Japan

Graph Minor project by Robertson and Seymour is perhaps the deepest theory in Graph Theory (and Combinatorics). It gives a deep structural characterization of graphs without any graph H as a minor. It also gives many exciting algorithmic consequences.

In this work, I would like to talk about our attempt to extend Graph minor project to directed graphs. Topics include

- The directed grid theorem
- The directed flat theorem
- The canonical tree decomposition
- The directed disjoint paths problem

Joint work with Stephan Kreutzer, O-joung Kwon, Archontia Giannopoulou.

Keywords: directed grid minor, directed disjoint paths, tree decomposition

An M-function associated with Goldbach's problem

Kohji Matsumoto

Graduate School of Mathematics, Nagoya University, Japan

Let

$$r_2(n) = \sum_{l+m=n} \Lambda(l)\Lambda(m),$$

where $\Lambda(\cdot)$ denotes the von Mangoldt function. Goldbach's classical conjecture would imply $r_2(n) > 0$ for all even $n \geq 6$. Hardy and Littlewood conjectured that $r_2(n) \sim nS_2(n)$ as even $n \to \infty$, where

$$S_2(n) = \prod_{p|n}\left(1 + \frac{1}{p-1}\right)\prod_{p\nmid n}\left(1 - \frac{1}{(p-1)^2}\right).$$

In view of this conjecture, it is interesting to evaluate the sum

$$A(x) = \sum_{n\leq x}(r_2(n) - nS_2(n)).$$

We assume the RH for the Riemann zeta-function $\zeta(s)$ hereafter. Let $\rho = 1/2+i\gamma$ be a non-trivial zero of $\zeta(s)$, and define, for $x > 0$,

$$\Psi(x) = \sum_{\gamma>0}\frac{x^{i\gamma}}{(1/2+i\gamma)(3/2+i\gamma)} = \sum_{m=1}^{\infty}\frac{x^{i\gamma_m}}{(1/2+i\gamma_m)(3/2+i\gamma_m)}, \tag{1}$$

where $0 < \gamma_1 < \gamma_2 < \cdots$. Then Fujii [4] proved the asymptotic formula

$$A(x) = -4x^{3/2}\cdot\Re\Psi(x) + O((x\log x)^{4/3}). \tag{2}$$

(A recent study related with Fujii's work can be found in [1].)

In view of (2), our main interest now becomes to study the behaviour of the quantity $\Psi(x)$. Let

$$f(\alpha) = \Psi(e^\alpha) \qquad (\alpha \in \mathbb{R}). \tag{3}$$

In [5], Fujii studied the value-distribution of $f(\alpha)$, and proved the following limit theorem. Assume that γ's are linearly independent over $\mathbb{Q}$ (which we call the LIC). Then Fujii stated the existence of the "density function" $F(x+iy)$ for which

$$\lim_{X\to\infty}\frac{1}{X}\mathrm{meas}\{0\leq\alpha\leq X \mid f(\alpha)\in R\} = \int\int_R F(x+iy)dxdy \tag{4}$$

holds for any rectangle R in $\mathbb{C}$. There is only a brief indication for the proof; Fujii claimed that the above result can be shown as an analogue of the result of Bohr and Jessen [3] for the value-distribution of $\zeta(s)$, and he wrote explicitly how to construct $F(x+iy)$, following the method of Bohr and Jessen [2].

The aim of this talk is to present the following "limit theorem", which is a generalization of Fujii's (4) in the framework of the theory of M-functions.

Theorem 1. We assume the RH and the LIC. There exists an explicitly constructed density function (M-function) $M : \mathbb{C} \to \mathbb{R}_{\geq 0}$, for which

$$\lim_{X\to\infty} \frac{1}{X} \int_0^X \Phi(f(\alpha))d\alpha = \int_{\mathbb{C}} M(w)\Phi(w)|dw| \tag{5}$$

holds for any test function $\Phi : \mathbb{C} \to \mathbb{C}$ which is continuous, or which is the characteristic function of either a compact subset of $\mathbb{C}$ or the complement of such a subset. The function $M(w)$ is continuous, tends to 0 when $|w| \to \infty$, $M(\overline{w}) = M(w)$, and

$$\int_{\mathbb{C}} M(w)|dw| = 1. \tag{6}$$

Note: Choosing $\Phi = \mathbf{1}_R$, we recover Fujii's result (4).

The name "M-function" is due to Ihara [6], but the origin of the theory goes back to the aforementioned work of Bohr and Jessen. In fact, let $\sigma > 1/2$, $T > 0$, R an arbitrary rectangle in $\mathbb{C}$ with the edges parallel to the axes. Let $V_\sigma(T,R)$ be the 1-dimensional Lebesgue measure of the set of all $\tau \in [-T,T]$ for which $\log\zeta(\sigma+i\tau) \in R$ holds (under a certain fixed choice of the branch). Then Bohr and Jessen proved that there exists a continuous non-negative function $\mathcal{F}_\sigma(\cdot)$, for which

$$\lim_{T\to\infty} \frac{1}{2T} V_\sigma(T,R) = \int_R \mathcal{F}_\sigma(w)|dw| \tag{7}$$

holds for any R, where $|dw| = (2\pi)^{-1}dudv$ ($w = u+iv$). This $\mathcal{F}_\sigma(\cdot)$ is the M-function for $\zeta(s)$.

The above (7) is the result on the average in "t-aspect". When we study more general zeta and L-functions, various other aspects may be considered. As for the average in "χ-aspect" of Dirichlet L-functions, the theory of M-functions was developed by [6], [7], [8] and [9]. Later, M-functions have been discovered in connection with the value-distribution of various other zeta and L-functions. Related articles were written by Mourtada-Murty, Akbary-Hamieh, Lebacque-Zykin, Matsumoto-Umegaki, Mine, and so on. A short survey can be found in [10].

The above theorem is a simpler version of M-functions, because we can prove this theorem as an analogue of the absolutely convergent case in [6] and [7]. In fact, our Theorem 1 is an analogue of [7, Theorem 4.2]. Still, however, I believe that this theorem gives an interesting new example of M-functions.

Keywords: Goldbach's problem, M-function, value-distribution

References

[1] G. Bhowmik, K. Halupczok, K. Matsumoto and Y. Suzuki, Goldbach representations in arithmetic progressions and zeros of Dirichlet L-functions, Mathematika **65** (2019), 57-97.

[2] H. Bohr and B. Jessen, Om Sandsynlighedsfordelinger ved Addition af konvekse Kurver, Den. Vid. Selsk. Skr. Nat. Math. Afd. (8) **12** (1929), 1-82.

[3] H. Bohr and B. Jessen, Über die Werteverteilung der Riemannschen Zetafunktion, Acta Math. **54** (1930), 1-35.

[4] A. Fujii, An additive problem of prime numbers II, Proc. Japan Acad. **67A** (1991), 248-252.

[5] A. Fujii, An additive problem of prime numbers III, Proc. Japan Acad. **67A** (1991), 278-283.

[6] Y. Ihara, On "M-functions" closely related to the distribution of L'/L-values, Publ. RIMS Kyoto Univ. **44** (2008), 893-954.

[7] Y. Ihara and K. Matsumoto, On certain mean values and the value-distribution of logarithms of Dirichlet L-functions, Quart. J. Math. (Oxford) **62** (2011), 637-677.

[8] Y. Ihara and K. Matsumoto, On $\log L$ and L'/L for L-functions and the associated "M-functions": Connections in optimal cases, Moscow Math. J. **11** (2011), 73-111.

[9] Y. Ihara and K. Matsumoto, On the value-distribution of logarithmic derivatives of Dirichlet L-functions, in "Analytic Number Theory, Approximation Theory and Special Functions", in Honor of H. M. Srivastava, Springer, 2014, pp. 79-91.

[10] K. Matsumoto and Y. Umegaki, On the value-distribution of symmetric power L-functions, to appear in Proc. Intern. Conf. on Number Theory, Dec 2017, Indian Inst. Tech. Ropar.

Not always 'Gentle and Clean'

Neal Koblitz

University of Washington, USA

H. Hardy's famous statement about number theory applies broadly to theoretical mathematics, but matters get much messier in applied mathematics, including applications of number theory. First, I will briefly describe two examples connected with the COVID-19 pandemic: some dubious mathematical modeling of the spread of the pandemic, and the rather unsuccessful application of cryptography to contact tracing. Then I will talk about Shor's algorithm for factoring integers, which is the most widely advertised selling point for quantum computation, and discuss its practicality. I'll end with some observations about our ability to predict future technology.

On some families of binary forms and the integers they represent

Michel Waldschmidt

Sorbonne University, France

The study of quadratic forms is a classical topic in number theory, going back to Fermat, Lagrange, Legendre and Gauss. According to Landau and Ramanujan, the number of positive integers $\leq N$ which are sums of two squares is asymptotically $\mathsf{C}_{\Phi_4} N(\log N)^{-\frac{1}{2}}$, where

$$\mathsf{C}_{\Phi_4} = \frac{1}{2^{\frac{1}{2}}} \cdot \prod_{p \equiv 3 \bmod 4} \left(1 - \frac{1}{p^2}\right)^{-\frac{1}{2}} = 0.764\,223\,653\,589\,220\ \ldots$$

is the so–called Landau–Ramanujan constant. More generally, P. Bernays [B] proved that, given a quadratic form $F(X,Y) = aX^2+bXY+cY^2 \in \mathbb{Z}[X,Y]$ with nonsquare positive discriminant b^2-4ac, there exists a positive constant C_F such that, for $N \to \infty$, the number of positive integers $m \in \mathbb{Z}$, $m \leq N$ which are represented by F is asymptotically $\mathsf{C}_F N(\log N)^{-\frac{1}{2}}$.

It is remarkable that a similar result for forms of higher degree was proved in full generality only recently, by Stewart and Xiao [SX], as follows. Let F be a binary form of degree $d \geq 3$ with non-zero discriminant. There exists $\mathsf{C}_F > 0$ and $\beta_d < \frac{2}{d}$ such that for $N \to \infty$, the number $R_F(N)$ of integers of absolute value at most N which are represented by $F(X,Y)$ satisfies

$$R_F(N) = \mathsf{C}_F N^{\frac{2}{d}} + O(N^{\beta_d}).$$

The number $\mathsf{C}_F = W_F \mathsf{A}_F$ is the product of W_F, a positive rational number which depends on the group of automorphisms of F, and A_F, the area of the fundamental domain $\{(x,y) \in \mathbb{R}^2 \mid F(x,y) \leq 1\}$.

Our main goal is to investigate the number of integers represented by at least one member in a given family of binary forms. The first family that we considered with Fouvry and Levesque [EFW] is the family of cyclotomic forms.

Recall the cyclotomic polynomials, defined by induction:

$$\phi_1(t) = t-1, \qquad t^n - 1 = \prod_{d|n} \phi_d(t), \qquad \phi_n(t) = \frac{t^n-1}{\displaystyle\prod_{d \neq n d|n} \phi_d(t)}\cdot$$

For $n \geq 1$, define

$$\Phi_n(X,Y) = Y^{\varphi(n)}\phi_n(X/Y).$$

This is a binary form in $\mathbb{Z}[X,Y]$ of degree $\varphi(n)$, where φ is Euler totient function.

The result of [SX] gives, for the number $R_{\Phi_n}(N)$ of integers $m \leq N$ represented by Φ_n for a given n with $\varphi(n) = d \geq 4$,

$$R_{\Phi_n}(N) = \mathsf{C}_{\Phi_n} N^{\frac{2}{d}} + O_\epsilon(N^{\beta_d+\epsilon}) \quad \text{with} \quad \mathsf{C}_{\Phi_n} = w_n \mathsf{A}_{\Phi_n}.$$

Here

$$\beta_d = \begin{cases} \frac{3}{d\sqrt{d}} & \text{for } d = 4, 6, 8, \\ \frac{1}{d} & \text{for } d \geq 10 \end{cases} \qquad \text{and} \quad \mathsf{A}_{\Phi_n} = \iint_{\Phi_n(x,y)\leq 1} \mathrm{d}x\mathrm{d}y.$$

The group of automorphisms of Φ_n is isomorphic either to the dihedral group $\mathbb{D}_2$ with 4 elements or to the dihedral group $\mathbb{D}_4$ with 8 elements:

$$\mathrm{Aut}\,\Phi_n = \begin{cases} \mathbb{D}_4 & \text{if 4 divides } n, \\ \mathbb{D}_2 & \text{otherwise,} \end{cases} \qquad w_n = \begin{cases} \frac{1}{8} & \text{if 4 divides } n, \\ \frac{1}{4} & \text{otherwise.} \end{cases}$$

The cyclotomic fundamental domain of the binary form Φ_n is

$$\mathcal{O}_n = \{(x,y) \in \mathbb{R}^2 \;\mid\; \Phi_n(x,y) \leq 1\}.$$

Its area A_{Φ_n} satisfies

$$\lim_{n\to\infty} \mathsf{A}_{\Phi_n} = 4.$$

Our first result is

Theorem 1 [FLW]. *The number of integers $m \leq N$ which are represented by at least one of the binary cyclotomic forms $\Phi_n(X,Y)$ with $n \geq 3$ is asymptotically*

$$\alpha \frac{N}{(\log N)^{\frac{1}{2}}} - \beta \frac{N}{(\log N)^{\frac{3}{4}}} + O\left(\frac{N}{\log N}\right)$$

as $N \to \infty$.

The main term

$$\alpha \frac{N}{\sqrt{\log N}} \quad \text{with} \quad \alpha = \mathsf{C}_{\Phi_4} + \mathsf{C}_{\Phi_3} = 1.403\,133\,059\,034\ \ldots$$

occurs from the contributions of the quadratic forms Φ_4 and Φ_3. The next term

$$-\beta \frac{N}{(\log N)^{\frac{3}{4}}} \quad \text{with} \quad \beta = 0.302\,316\,142\,357\ \ldots$$

occurs from the contribution of the numbers which are represented by the form Φ_4 and also by the form Φ_3.

The error term is sharp; it takes into account all binary cyclotomic forms of degree ≥ 4.

Theorem 1 deals with the family of cyclotomic forms of degree ≥ 2. More generally, given an integer $d \geq 2$, we can investigate the same question for the family of cyclotomic forms of degree $\geq d$. Let us call *totient* a positive integer which is a value of Euler totient function φ. For $d \geq 4$, let $d^\dagger$ denote the next totient $> d$.

Theorem 2 [FW]. *Let $d \geq 4$ be a totient. As $N \to \infty$, the number $\mathcal{A}_{\geq d}(N)$ of integers $m \leq N$ for which there exist n and (x, y) with $\Phi_n(x, y) = m$, $\varphi(n) \geq d$ and* $\max\{|x|, |y|\} \geq 2$*, is asymptotically*

$$\mathcal{A}_{\geq d}(N) = C_d N^{\frac{2}{d}} + \begin{cases} O_\epsilon(N^{\frac{13}{32}+\epsilon}) & \textit{for } d = 4, \\ O(N^{\frac{2}{d^\dagger}}) & \textit{for } d \geq 6, \end{cases}$$

with

$$C_d = \sum_n \mathsf{C}_{\Phi_n},$$

where the sum is over the set of integers n such that $\varphi(n) = d$ and n is not congruent to 2 modulo 4.

If $d \geq 6$ and $d^\dagger = d + 2$, then the error term is sharp.

One main tool for the proof of Theorem 2 is the following variant of a result by Stewart and Xiao:

Theorem 3 [FW]. *Let F_1 and F_2 be two nonisomorphic binary forms of the same degree $d \geq 3$. Assume that their discriminants are nonzero. Then for any $\varepsilon > 0$ we have*

$$\mathcal{N}_{F_1,F_2}(B) = O(B^{\gamma_d+\varepsilon}),$$

with

$$\gamma_d = \begin{cases} \frac{2}{3} + \frac{73}{36\sqrt{3}} & \textit{if } d = 3, \\ \frac{1}{2} + \frac{9}{4\sqrt{d}} & \textit{if } 4 \leq d \leq 20, \\ 1 & \textit{for } d \geq 21. \end{cases}$$

References

[B] P. Bernays, *Über die Darstellung von positiven, ganzen Zahlen durch die primitiven, binären quadratischen Formen einer nicht quadratischen Diskriminante*, Ph.D. dissertation, Georg-August-Universität, Göttingen, Germany, 1912.
`http://www.ethlife.ethz.ch/archive_articles/120907_bernays_fm/`

[FLW] E. Fouvry, C. Levesque & M. Waldschmidt, *Representation of integers by cyclotomic binary forms*, Acta Arith., **184**.1 (2018), 67 - 86.
`DOI: 10.4064/aa171012-24-12` `arXiv: 712.09019`

[FW] E. Fouvry & M. Waldschmidt, *Sur la représentation des entiers par des formes cyclotomiques de grand degré*, Bull. Soc. Math. France, **148**.2 (2020), 253–282.
`DOI 10.24033/bsmf.2805`

[SX] C.L. Stewart & S.Y. Xiao, *On the representation of integers by binary forms*, Math. Ann. **375** (2019), 133–163.
`DOI 10.1007/s00208-019-01855-y` `arXiv: 1605.03427, v2 March 2018`

Computing Igusa's local zeta function of univariates in deterministic polynomial-time

Nitin Saxena

Indian Institute of Technology Kanpur, India

Igusa's local zeta function $Z_{f,p}(s)$ is the generating function that counts the number of integral roots, $N_k(f)$, of $f(x) \mod p^k$, for all k. It is a famous result, in analytic number theory, that $Z_{f,p}$ is a rational function in $\mathbb{Q}(p^s)$. We give an elementary proof of this fact for a univariate polynomial f. Our proof is constructive as it gives a closed-form expression for the number of roots $N_k(f)$.

Our proof, when combined with the recent root-counting algorithm of (Dwivedi, Mittal, Saxena, CCC, 2019), yields the first poly$(|f|, \log p)$ time algorithm to compute $Z_{f,p}(s)$.

Previously, an algorithm was known only in the case when f completely splits over $\mathbb{Q}p$; it required the rational roots to use the concept of generating function of a tree (Zúñiga-Galindo, J.Int.Seq., 2003).

The talk is based on the joint work with Ashish Dwivedi, published in 14th Biannual Algorithmic Number Theory Symposium, ANTS-XIV, 2020. Additionally, I will overview the earlier methods, and state the major open questions, in the area of factoring modulo prime-powers.

Keywords: Igusa, local, zeta function, discriminant, valuation, deterministic, root, counting, modulo, prime-power .

From automata to permutation groups

Peter J. Cameron

University of St Andrews, UK

In this talk I will take a quick trip through various related areas of discrete mathematics, including automata theory, transformation semigroups, permutation groups, and Latin squares, to illustrate some recent developments showing the links between them.

An *automaton* here is a very simple machine. It has a number of internal states, and when it reads a symbol it changes its state. If there is a sequence of symbols such that reading them in turn brings the machine into a fixed state regardless of where it started, the automaton is said to be *synchronizing*, and the sequence is called a *reset word*.

One of the oldest problems in automata theory, still unsolved, is the infamous *Černý conjecture* [4], asserting that if an n-state automaton is synchronizing, then it has a reset word of length at most $(n-1)^2$.

Each letter in its alphabet can be thought of as indexing a transformation on the set Ω of states. Since we are allowed to compose these basic transitions, we can think of an automaton as a transformation monoid on Ω with a prescribed set of generators. It is synchronizing if the monoid contains an element of *rank* 1 (that is, whose image has cardinality 1. This allows us to make a link to transformation monoids and permutation groups.

I will give a brief introduction to the theory of permutation groups, sketching the reduction to *primitive groups*. The structure of primitive permutation groups is described by the *O'Nan–Scott Theorem* [12], which allows the *Classification of Finite Simple Groups* (CFSG) to be brought to bear on our problems. The O'Nan–Scott theorem divides primitive groups into four main classes. One of these consists of *almost simple groups*, which have to be dealt with individually. Two of the other three, *affine groups* and *wreath products*, are automorphism groups of well-studied structures (affine spaces and Cartesian lattices respectively).

Recently [2], with Rosemary Bailey, Cheryl Praeger and Csaba Schneider, I have looked at the remaining type, the *diagonal groups*, and produced both a descriptive and an axiomatic study of the geometries which have diagonal groups as their automorphism groups. In the two-dimensional case, these geometries are just *Latin squares*, re-interpreted in terms of partitions. But in higher dimensions the group emerges naturally from the combinatorial assumptions. Indeed, in three dimensions, we have one type of *Latin cube* (from several types which have been studied [11]), and show that such Latin cubes are 3-dimensional analogues of *Cayley tables* of groups.

Returning to permutation groups, since a permutation group (regarded as a transformation monoid in its own right) cannot be synchronizing in the previous sense, we re-interpret the term and say that the group G is *synchronizing* if, for any transformation f on Ω which is not a permutation, the monoid generated by G and f is synchronizing in the previous sense [1].

The class of synchronizing groups lies strictly between the primitive groups and the 2-*transitive* groups, and poses some very interesting questions: how do we recognise a synchronizing group? and how do the O'Nan–Scott classes look in this classification?

In the 1950s, Hall and Paige [7] conjectured that the Cayley table of a finite group G has an orthogonal mate if and only if the Sylow 2-subgroups of G are either trivial or of odd order. They showed the necessity of the condition; its sufficiency was proved in 2009 by Stuart Wilcox, Anthony Evans and John Bray, using the Classification of Finite Simple Groups [3,5,14].

Using this, it is possible to draw our threads together by showing that every diagonal group in dimension at least 2 is non-synchronizing.

Brief mention will be made of other topics: clique number and chromatic number of highly symmetric graphs; strongly regular graphs; Hamming graphs; recognising Cayley tables of groups as Latin squares; connections with experimental design in statistics; etc. Further information about these can be found in the references.

Keywords: automaton, transformation monoid, permutation group, primitive permutation group, O'Nan–Scott Theorem, Classification of Finite Simple Groups, Latin squares, Latin cubes, Hall–Paige conjecture

References

[1] J. Araújo, P. J. Cameron and B. Steinberg, Between primitive and 2-transitive: Synchronization and its friends, *Europ. Math. Soc. Surveys* **4** (2017), 101–184.

[2] R. A. Bailey, Peter J. Cameron, Cheryl E. Praeger and Csaba Schneider, The geometry of diagonal groups, arXiv `2007.10726`.

[3] J. N. Bray, Q. Cai, P. J. Cameron, P. Spiga and H. Zhang, The Hall-–Paige conjecture, and synchronization for affine and diagonal groups, *J. Algebra* **545** (2020), 27–42.

[4] J. Černý, Poznámka k homogénnym experimentom s konečnými automatmi, *Matematicko-fyzikálny časopis Slovenskej Akadémie Vied* **14** (1964), 208–216.

[5] Anthony B. Evans, The admissibility of sporadic simple groups, *J. Algebra* **321** (2009), no. 1, 105–116.

[6] M. Frolov, Recherches sur les permutations carrées, *J. Math. Spéc.* (3) **4** (1890), 8–11.

[7] Marshall Hall Jr. and L. J. Paige, Complete mappings of finite groups, *Pacific J. Math.* **5** (1955), 541–549.

[8] J. A. Nelder, The analysis of randomized experiments with orthogonal block structure, I: Block structure and the null analysis of variance, *Proceedings of the Royal Society of London, Series A* **283** (1965), 147–162.

[9] Peter M. Neumann, Charles C. Sims and James Wiegold, Counterexamples to a theorem of Cauchy, *J. London Math. Soc.* (1) **43** (1968), 234.

[10] C. E. Praeger and C. Schneider, *Permutation Groups and Cartesian Decompositions*, London Math. Soc. Lecture Notes **449**, Cambridge University Press, Cambridge, 2018.

[11] D. A. Preece, Latin squares, Latin cubes, Latin rectangles, etc., In *Encyclopedia of Statistical Sciences*, Volume 4 (eds. S. Kotz and N. L. Johnson), Wiley: New York, 1983, pp. 504–510.

[12] L. L. Scott, Representations in characteristic p, In *The Santa Cruz Conference on Finite Groups* (Univ. California, Santa Cruz, Calif., 1979), Proc. Sympos. Pure Math., vol. 37, Amer. Math. Soc., Providence, R.I., 1980, pp. 319–331.

[13] S. S. Shrikhande, The uniqueness of the L_2 association scheme, *Ann. Math. Statist.* **30** (1959), 781–798.

[14] Stewart Wilcox, Reduction of the Hall–Paige conjecture to sporadic simple groups, *J. Algebra* **321** (2009), no. 5, 1407–1428.

Gap sets for the spectra of cubic graphs

Peter Sarnak

Institute of Advanced Study, USA

The spectra of large locally uniform geometries have been studied widely and from different points of view, including in applications .They include Ramanujan Graphs and Buildings, euclidean and hyperbolic spaces and more general locally symmetric spaces. We review some of these briefly highlighting rigidity features.We then focus on the simplest case of finite cubic graphs which prove to be surprisingly rich. As one imposes restrictions on these graphs, planarity, fullerenes, their spectra become rigid.

Joint work with Alicia Kollar and Fan Wei .

Recent results on Cheeger-type inequalities for Cayley graphs

Prasad Tetali

Georgia Institute of Technology, USA

It is well-known, particularly since the introduction of Ramanujan expanders, that in d-regular graphs (say) the separation of the non-trivial eigenvalues from -d and d can be quantified using appropriate isoperimetric constants defined using edge and vertex boundaries of sets of vertices. The speaker will share such results obtained recently in the context of Cayley graphs.

The product of three primes in Arithmetic progressions

R. Balasubramanian

Institute of Mathematical Science, India

A conjecture due to Erdös, Odlyzko and Sarközy predicts that, given two coprime integers a and q, one can find primes p_1 and p_2, both less than q such that $p_1p_2 \equiv a \bmod(q)$.

A weaker conjecture would be : Given q, there exists $f(q)$ such that for any a coprime to q one can find three primes p_1, p_2, p_3, all less than $f(q)$, such that $p_1p_2 \equiv a \bmod(q)$. It was proved by Olivier Ramaré and Priyamvad Srivastav that the above is true with $f(q) = (650q)^3$. The result is obtained by analytic tools (Brun Titchmarsh theorem) and additive combinatorics (Knesers therem) In this talk, we shall indicate the proof of the theorem and the ongoing work (with Ramaré and Priyamvad) on how to improve the analytic part and combinatorial part in the proof.

Keywords: Primes in arithmetic progression, least prime quadratic residue, Linnik's theorem

Graph density inequalities and sums of squares

G. Blekherman, A. Raymond, M. Singh and R.R. Thomas

University of Washington, USA

Many results in extremal graph theory can be formulated as inequalities on graph densities. While many inequalities are known, many more are conjectured. A standard tool to establish an inequality is to write the expression whose nonnegativity needs to be certified as a sum of squares. This technique has had many successes but also limitations. In this talk I will describe new restrictions that show that several simple inequalities cannot be certified by sums of squares. These results extend to the powerful frameworks of flag algebras by Razborov and graph algebras by Lovasz and Szegedy.

Keywords: extremal graph theory, sum of squares, graph density inequalities

Higher rank exponential sums

Ritabrata Munshi

Indian Statistical Institute, Kolkata, India

Exponential sums,

$$\sum_{N<n\leq 2N} e^{2\pi i P(n)}$$

with P a real valued function, play a central role in analytic number theory. Several problems in analytic number theory, e.g. bounding the size of the Riemann zeta function, the Gauss circle problem, the Dirichlet divisor problem, ultimately boil down to establishing nontrivial bounds for such sums. The methods of Weyl, van der Corput and Vinogradov are usually employed to study such sums. One advantage of these classical methods is the fact that they are effective even when the length of the sum is small. The higher rank analogues of exponential sums are formed by twisting the above sum by Fourier-Whittaker coefficients of automorphic forms. Suppose f is a Hecke Maass or modular form with normalised Fourier coefficients $\lambda_f(n)$, then we consider the sum

$$\sum_{N<n\leq 2N} \lambda_f(n) e^{2\pi i P(n)}.$$

Though such sums are expected to have nontrivial cancellation even when the length is small, at present we can prove this only when the length is sufficiently large compared to the derivatives of the function P, and the conductor of f. In particular, one can show that

$$\sum_{N<n\leq 2N} \lambda_f(n) e^{2\pi i \alpha n^\beta} \ll_f N^{1-\delta},$$

where δ is a function of β, and $\delta > 0$ as long as $\beta < 3/2$. It is a challenging problem to extend the range beyond $\beta = 3/2$. Much less is known at higher rank. For f an automorphic form for $GL_d(\mathbb{A}_{\mathbb{Q}})$ with normalised Fourier-Whittaker coefficients $\lambda_f(n_1, \ldots, n_{d-1})$, one can prove

$$\sum_{N<n\leq 2N} \lambda_f(n, 1, \ldots, 1) e^{2\pi i \alpha n^\beta} \ll_f N^{1-\delta},$$

with $\delta > 0$ as long as $\beta < 2/d$. This can be called the convexity range as it corresponds to the convexity bound for the corresponding L-function. For $d = 3$ however, as has been recently shown, one can go beyond this range and get a nontrivial bound for $\beta < 5/7$.

Keywords: Exponential sums, nonlinear twists

1/2-Conjectures on the domination game

Sandi Klavžar

Faculty of Mathematics and Physics, University of Ljubljana, Slovenia

Faculty of Natural Sciences and Mathematics, University of Maribor, Slovenia

Institute of Mathematics, Physics and Mechanics, Ljubljana

The domination game [1] is played on an arbitrary graph G by two players, Dominator and Staller. They are taking turns choosing a vertex from G such that at least one previously undominated vertex becomes dominated. The game ends when no move is possible; then the score of the game is the total number of vertices chosen. Dominator wants to minimize the score, while Staller wants to maximize it. The number of vertices selected in a game in which Dominator is the first to select a vertex, and both players follow optimal strategies, is the *game domination number* $\gamma_g(G)$ of G.

A central theme in the investigation of the game domination number are upper bounds on it in terms of the order $n(G)$ of a graph G. It all started with the 3/5-conjecture [9] asserting that if G is an isolate-free graph, then $\gamma_g(G) \leq \frac{3}{5}n(G)$. A strong support for the conjecture was provided in [6] where it was proved that it is true for all graphs with minimum degree at least 2. The conjecture is still open in general, the best upper bound recently proved in [2] asserts that if G does not contain isolated vertices, then $\gamma_g(G) \leq \frac{5}{8}n(G)$.

Recall that a traceable graph is a graph that contains a Hamiltonian path. Then another appealing conjecture, first stated in [7], reads as follows.

Conjecture 1 (Rall's 1/2-conjecture) *If G is a traceable graph, then*

$$\gamma_g(G) \leq \left\lceil \frac{1}{2} n(G) \right\rceil .$$

Some support for the conjecture was first given in [7, 8].

A graph G is a 1/2-*graph* if G is traceable and $\gamma_g(G) = \lceil n(G)/2 \rceil$, that is, 1/2-graphs are the traceable graphs that attain the equality in Conjecture 1. Rall's conjecture was in part motivated by the following non-trivial result (see [10]) for paths P_n, $n \geq 1$, and cycles C_n, $n \geq 3$:

$$\gamma_g(P_n) = \gamma_g(C_n) = \begin{cases} \lceil \frac{n}{2} \rceil - 1; & n \equiv 3 \pmod 4, \\ \\ \lceil \frac{n}{2} \rceil; & \text{otherwise.} \end{cases}$$

This result implies that each of P_n and C_n is a $1/2$-graph if and only if $n \pmod 4 \in \{0, 1, 2\}$.

Very recently, in [3], the following general 1/2-conjecture has been posed.

Conjecture 2 *If $\delta(G) \geq 2$, then*

$$\gamma_g(G) \leq \left\lceil \frac{1}{2} n(G) \right\rceil .$$

In this talk, efforts from [3–5] to (dis)prove Conjectures 1 and 2 will be presented. In this extended abstract, we list the following selected results.

In [4] several additional families of graphs that support Conjecture 1 were determined, in particular, it was proved that the conjecture holds for all unicyclic (traceable) graphs. Extensive related computer experiments were also performed.

Proposition 1 ([4]) *(i) If $4 \leq n \leq 21$, and G is a path P_n with two additional edges, then $\gamma_g(G) \leq \lceil \frac{n}{2} \rceil$. If $4 \leq n \leq 15$, the same holds for a path P_n with three additional edges.*

(ii) If $4 \leq n \leq 24$ and G is a cycle C_n with two additional edges, then $\gamma_g(G) \leq \gamma_g(C_n) \leq \lceil \frac{n}{2} \rceil$. If $4 \leq n \leq 20$, the same holds for a cycle C_n with three additional edges.

Theorem 1 ([5]) *If G is a graph with* $\mathrm{diam}(G) = 2$, *then*

$$\gamma_g(G) \leq \left\lceil \frac{n(G)}{2} \right\rceil .$$

Moreover, the equality holds if and only if G is one of the graphs from Fig. 1 or the Petersen graph.

Theorem 2 ([3]) *If G is a claw-free cubic graph, then $\gamma_g(G) \leq \frac{n(G)}{2}$.*

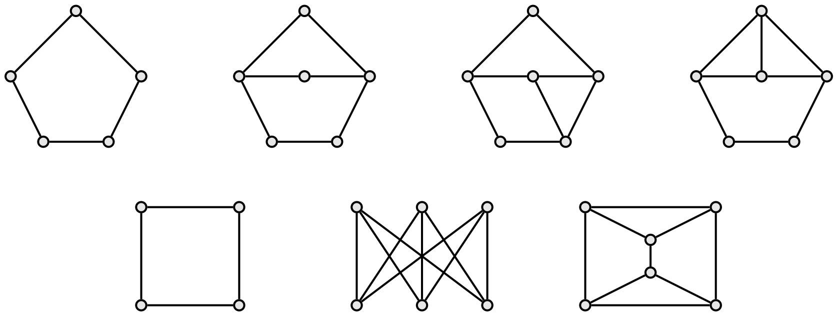

Figure 1: Sporadic graphs with $\gamma_g(G) = \lceil n(G)/2 \rceil$.

Theorem 3 ([3]) *If G is a traceable line graph, then* $\gamma_g(G) \leq \left\lceil \frac{n(G)}{2} \right\rceil$.

Finally, provided that a counterexample to Conjecture 2 will be found, the following alternative, weaker versions of Conjecture 2 could become interesting.

Conjecture 3 *If $\delta(G) \geq 2$, then $\gamma_g(G) \leq \frac{n(G)}{2} + C$, where C is a universal constant.*

Conjecture 4 *There exists a constant $c < \frac{3}{5}$ such that every graph G with $\delta(G) \geq 2$ and $n(G) \geq 6$ satisfies $\gamma_g(G) \leq c \cdot n(G)$.*

Keywords: domination game; Rall's 1/2-conjecture; 1/2-conjecture

References

[1] B. Brešar, S. Klavžar, D.F. Rall, Domination game and an imagination strategy, SIAM J. Discrete Math. 24 (2010) 979–991.

[2] Cs. Bujtás, General upper bounds on the game domination number, Discrete Appl. Math. 285 (2020) 530–538.

[3] Cs. Bujtás, V. Iršič, S. Klavžar, 1/2-conjectures on the domination game and claw-free graphs, arXiv:2010.14273 [math.CO] (27 Oct 2020).

[4] Cs. Bujtás, V. Iršič, S. Klavžar, K. Xu, On Rall's 1/2-conjecture on the domination game, Quaest. Math. (2020) DOI:10.2989/16073606.2020.1822945.

[5] Cs. Bujtás, V. Iršič, S. Klavžar, K. Xu, The domination game played on diameter 2 graphs, arXiv:2009.09760 [math.CO] (21 Sep 2020).

[6] M.A. Henning, W.B. Kinnersley, Domination game: a proof of the 3/5-conjecture for graphs with minimum degree at least two, SIAM J. Discrete Math. 30 (2016) 20–35.

[7] T. James, S. Klavžar, A. Vijayakumar, The domination game on split graphs, Bull. Aust. Math. Soc. 99 (2019) 327–337.

[8] S. Klavžar, D.F. Rall, Domination game and minimal edge cuts, Discrete Math. 342 (2019) 951–958.

[9] W.B. Kinnersley, D.B. West, R. Zamani, Extremal problems for game domination number, SIAM J. Discrete Math. 27 (2013) 2090–2107.

[10] G. Košmrlj, Domination game on paths and cycles, Ars Math. Contemp. 13 (2017) 125–136.

On the harmonic volume of Fermat curves

Payman Eskandari[1] and V. Kumar Murty[2]

[1]Department of Mathematics, University of Toronto, Canada,

[2]Fields Institute for Research in Mathematical Sciences Canada

Let X be a smooth, projective curve over the complex numbers and denote by $Jac(X)$ its Jacobian. Fix a base point $e \in X$ and consider the Albanese map $X \longrightarrow Jac(X)$ with this base point. Denote by X_e the image of X under this map. The Ceresa cycle C is defined by $X_e - (-1)_* X_e$ and it is homologically trivial. Thus, it is the boundary of some chain, C, say and we can define a functional on certain differential forms by integrating along C. This gives an element in the (Griffiths) intermediate Jacobian via the Abel-Jacobi map.

We prove that if X is the Fermat curve $x^n + y^n = z^n$ where n has a prime divisor greater than 7, then the Abel-Jacobi image of the Ceresa cycle is of infinite order. This is connected to the concept of harmonic volume introduced by Bruno Harris which is defined in terms of Chen's iterated integrals.

In this talk, we will explain all of the background concepts and introduce the main result.

Keywords: Ceresa cycle, Abel-Jacobi map, harmonic volume, Fermat curve

Algebraic independence and modular forms

Yuri Nesterenko

Lomonosov Moscow State University, Russia

Theta function $\theta(z,\tau)$ is defined by the series $\theta(z,\tau)=\sum_{n\in\mathbb{Z}} e^{2\pi i n^2\tau+2\pi i n z}$. Let's denote by the letter $\mathcal{K}$ least algebraically closed field containing functions

$$\tau,\ q=e^{\pi i\tau},\ \theta(0,\tau)=1+2\sum_{n=1}^{\infty} q^{n^2}, \qquad \Im\tau>0$$

and closed with respect of differentiation $D=\frac{1}{\pi i}\frac{d}{d\tau}=q\frac{d}{dq}$. Classical relations imply that theta-constants $\theta_2(\tau),\theta_3(\tau)=\theta(0,\tau),\theta_4(\tau)$ and their logarithmic derivatives

$$\psi_2(\tau)=\frac{D\theta_2}{\theta_2},\quad \psi_3(\tau)=\frac{D\theta_3}{\theta_3},\quad \psi_4(\tau)=\frac{D\theta_4}{\theta_4}.$$

belong to the field $\mathcal{K}$. It is not difficult to conclude that the field $\mathcal{K}$ contains the modular functions $\lambda(\tau)$, $j(\tau)$ and Eisenstein's series $E_{2k}(\tau)$ for any $k\geq 1$. Moreover for any $(a,b)\in\mathbb{Q}^2\setminus(0,0)$ we have $\theta(a\tau+b,\tau)\in\mathcal{K}$, $\wp(a\tau+b,1,\tau)\in\mathcal{K}$, where $\wp(z)$ is the Weierstrass elliptic function. Ramanujan introduced series

$$P(z)=1-\gamma_1\sum_{n=1}^{\infty}\sigma_1(n)z^n,\ Q(z)=1-\gamma_2\sum_{n=1}^{\infty}\sigma_3(n)z^n,\ R(z)=1-\gamma_3\sum_{n=1}^{\infty}\sigma_5(n)z^n,$$

where $\gamma_1=24,\gamma_2=-240,\gamma_3=504$, $\sigma_k(n)=\sum_{d|n}d^k$, and proved in 1916 that

$$\delta P=\frac{1}{12}(P^2-Q),\quad \delta Q=\frac{1}{3}(PQ-R),\quad \delta R=\frac{1}{2}(PR-Q^2),\quad \delta=z\frac{d}{dz}.$$

Eisenstein functions have a close connection to Ramanujan series $E_2(\tau)=P(e^{2\pi i\tau})$, $\quad E_4(\tau)=Q(e^{2\pi i\tau})$, $\quad E_6(\tau)=R(e^{2\pi i\tau})$, $\quad \mathrm{Im}\tau>0$, Due to the system of differential equations connecting functions P,Q,R we conclude that the field $\mathbb{C}(\tau,q,P(q^2),Q(q^2),R(q^2))$ is closed with respect of differentiation D. Consequently $\mathcal{K}$ is algebraic closure of this field. In 1969 K. Mahler proved that functions

$$\tau,\ q,\ j(\tau),\ Dj(\tau),\ D^2j(\tau)$$

are algebraically independent over $\mathbb{C}$. Since this functions belong to the field $\mathcal{K}$, we conclude that the transcendence degree of $\mathcal{K}$ over $\mathbb{C}$ equals to 5.

The variety of functions (elliptic, modular, hypergeometric)involved in this theory, and connected by different sometimes absolutely unexpected identities allowed to receive many results about transcendence and algebraic independence of values of these functions. Let's give some examples.

1. In 1935 K.Mahler and J. Popken proved that *for every* $\tau, \Im\tau > 0$ *at least one of numbers*

$$E_2(\tau), \qquad E_4(\tau), \qquad E_6(\tau) \tag{8}$$

is transcendental.

2. G. Chudnovskii, 1984: *Let* $\wp(z)$ *be Weierstrass elliptic function with invariants* g_2, g_3, *let* ω *be a nonzero period of* $\wp(z)$ *and* η *corresponding quasi-period. Then at least two of the numbers*

$$g_2,\ g_3,\ \frac{\omega}{\pi},\ \frac{\eta}{\pi}$$

are algebraically independent over $\mathbb{Q}$. Numbers ω and η can be expressed as values of hypergeometric functions. In case $y^2 = 4x^3 - 4x$ we have: *for any algebraic number* α, $0 < |\alpha| < 1$ *the numbers*

$$F\left(\frac{1}{2}, \frac{1}{2}, 1; \alpha\right), \qquad F'\left(\frac{1}{2}, \frac{1}{2}, 1; \alpha\right)$$

are algebraically independent over $\mathbb{Q}$. Chudnovskii's theorem is equivalent to the statement: *for every* $\tau, \Im\tau > 0$ *at least two of numbers* (8) *are algebraically independent.*

3. The following more general result was proved in 1996 by Nesterenko: *Let be* $\tau \in \mathbb{C}, \Im\tau > 0$. *Then at least three of the numbers*

$$e^{\pi i \tau}, E_2(\tau), E_4(\tau), E_6(\tau)$$

are algebraically independent over $\mathbb{Q}$. The following assertion is true: *For any natural number* d *the numbers*

$$\pi,\ e^{\pi\sqrt{d}},$$

are algebraically independent over $\mathbb{Q}$. The following corollary was proved by D. Bertrand,

Let $f(\tau)$ *be a non constant meromorphic modular form, defined over* $\mathbb{Q}$. *For all* $\alpha \in \mathbb{C}, \Im\alpha > 0$, *distinct from poles of* $f(\tau)$, *such that* $e^{2\pi i\alpha} \in \overline{\mathbb{Q}}$, *the numbers* $f(\alpha)$, $Df(\alpha)$ *and* $D^2 f(\alpha)$ *are algebraically independent over* $\mathbb{Q}$.

Invited Talks

Linear groups - some combinatorial themes

B. Sury

Indian Statistical Institute, Bangalore, India

Some combinatorial group theoretic properties lead to interesting characterizations of arithmetic groups and of some of their special properties. They are often studied using class field theory. Conversely, properties of Artin's reciprocity law such as their uniqueness essentially follow from certain group theoretic properties. Other purely number theoretic questions about polynomials can sometimes be naturally resolved using simple properties of linear groups. We discuss a few such applications.
Keywords: Arithmetic Groups, Bounded Generation, p-adic groups, Tits building, Reciprocity laws, Central extensions, Monodromy groups of polynomials

An upper bound for counting number fields

Frank Thorne

University of South Carolina, USA

How many number fields are there of fixed degree and bounded discriminant?

I will first give a brief overview of what is expected, and of some sharp results that can be proved in some special cases. I will then sketch a proof that for each $n \geq 6$ there are $\ll X^{1.6(\log n)^2}$ fields of degree n and discriminant $< X$. This improves upon work of Schmidt, Ellenberg-Venkatesh, and Couveignes, and is joint work with Robert Lemke Oliver.

Keywords: number field counitng, Malle's conjecture

Some analogues of Wilton's formula and their applications

Kalyan Chakraborty

Kerala School of Mathematics, India

J. R. Wilton obtained an expression for the product of two Riemann zeta functions. This expression played a crucial role to find the approximate functional equation for the product of two Riemann zeta functions in the critical region. In this talk, we will discuss some analogous expressions for the product of two Dedekind zeta functions. We will use these expressions to find some expressions for Dedekind zeta values attached to arbitrary real as well as imaginary quadratic number fields at any positive integer. If time permits, we will look at the analogous expressions for higher degree number fields. This is a joint work with Soumyarup Banerjee and Azizul Hoque.

Keywords: Wilton's formula, Dedekind zeta function, Special values of Dedekind zeta function, Nakajima dissection

On the Brumer-Stark conjecture

Mahesh Kakde

Indian Institute of Science, India

The conjecture of Brumer-Stark can either be stated as a statement about annihilation of an ideal class group or as a statement of existence of certain algebraic integer. In the talk I will sketch a proof of the Brumer-Stark conjecture away from $p = 2$. This is a joint work with Samit Dasgupta

Key Words: extremal graph theory, sum of squares, graph density inequalities

Recent results and open problems on CIS Graphs

Martin Milanič

FAMNIT and IAM, University of Primorska, Slovenia

Definition and historical remarks. A *clique* in a (finite, simple, undirected) graph is a set of pairwise adjacent vertices; a *stable set* is a set of pairwise non-adjacent vertices. A clique is *maximal* if it is not contained in any larger clique and *strong* if it intersects every inclusion-maximal stable set. Maximal and strong stable sets are defined analogously. A graph G is a *CIS graph* if every maximal clique C intersects every maximal stable set S, or, equivalently, if every maximal clique is strong. The study of CIS graphs is rooted in the 1960s in the work of Grillet [9], who characterized partially ordered sets in which each maximal chain intersects each maximal antichain. The acronym CIS was suggested by Andrade *et al.* [2].

Main properties, results, and questions. It is clear from the definition that a graph is CIS if and only if its complement is CIS. Furthermore, the class of CIS graphs is closed under substitution (see, e.g., [2]). However, understanding which graphs have the CIS property seems difficult. One of the possible reasons for this might be the fact that the class of CIS graphs is not closed under vertex deletion. This is related to the following necessary condition for the CIS property: each induced 4-vertex path $abcd$ in G extends to an induced *bull*, that is, some vertex v in G has to be adjacent to both b and c and non-adjacent to both a and d. More generally, in each CIS graph, for each $k \geq 2$, each induced copy of F_k or its complement is settled, where F_k is a non-CIS graph of order $2k$ consisting of a clique of size k, a stable set of size k and a perfect matching between them. An induced F_k (or its complement, which also consists of a clique of size k and a stable set of size k) in a graph G is *settled* if there is a vertex that is adjacent to all vertices in the clique and to no vertices in the stable set. In the 1990s (see [21]) Chvátal conjectured that a graph G not containing F_3 or its complement as induced subgraph is CIS if and only if each induced 4-vertex path in G is settled. This was proved by Deng *et al.* [7] and then independently by Andrade *et al.* [2].

Known particular classes of CIS graphs include graphs without an induced 4-

vertex path, line graphs of balanced complete bipartite graphs, and graphs in which every maximal clique is *simplicial*, that is, it consists of a vertex and all its neighbors. However, for general graphs, no good characterization or recognition algorithm for the CIS property is known. Recognizing CIS graphs is believed to be co-NP-complete [21], conjectured to be co-NP-complete [22], and conjectured to be polynomial-time solvable [2].

Solved special cases. Given two graphs F and G, we say that graph G is F-*free* if no induced subgraph of G is isomorphic to F. The problem of recognizing CIS graphs is known to be solvable in polynomial time for the classes of line graphs [4] and (more generally) claw-free graphs [1], comparability graphs (see [21]), graphs of bounded clique-width (by a meta-theorem of Courcelle *et al.* [6], using also [19]), and F-free graphs where F is any graph with at most 4 vertices, see [5]. In particular, denoting by C_4 the 4-vertex cycle, the fact that every strong clique in a C_4-free graph is simplicial (see [13]) implies that a C_4-free graph is CIS if and only if every maximal clique is simplicial. The problem of determining if a given F-free graph is CIS is also easily seen to be solvable in polynomial time if F is either the bull (since in this case no induced P_4 can be settled) or a complete graph (by enumerating all the maximal cliques, which are of bounded size, and verifying whether they are all strong).

Vertex-transitive CIS graphs. A graph G is *vertex-transitive* if, for any two vertices u and v, there is an automorphism of G mapping u to v. The *clique number* of a graph G, denoted by $\omega(G)$, is the maximum size of a clique, and its *stability number*, denoted by $\alpha(G)$, is the maximum size of a stable set. A graph is *well-covered* if all its maximal stable sets have the same size. Generalizing earlier work on circulant graphs by Boros *et al.* [3], the study of strong cliques and stable sets in the context of general vertex-transitive graphs was initiated by Dobson *et al.* [8]. They proved that a vertex-transitive graph G is a CIS graph if and only if it is well-covered, its complement is well-covered and $\alpha(G) \cdot \omega(G) = |V(G)|$. In particular, this means that vertex-transitive CIS graphs attain equality in the inequality $\alpha(G) \cdot \omega(G) \leq |V(G)|$, which is known to hold for every vertex-transitive graph. Furthermore, a recent result by Hujdurović [12] states that a vertex-transitive graph G is CIS if and only if G has a strong clique and a strong stable set.

An Erdős–Hajnal-type question. Vertex-transitive CIS graphs share the well-known property of perfect graphs, that the order of the graph is bounded from above by the product of its clique number and stability number. This fact motivated Dobson *et al.* [8] to ask whether this property holds for all CIS graphs. Alcón *et al.* [1] showed that this question has an affirmative answer for claw-free graphs, but not in general. However, as they noted, it is not known whether the class of CIS graphs has the *Erdős–Hajnal property*, that is, whether there exists a real number $r > 1$ such that every CIS graph G satisfies $|V(G)| \leq (\alpha(G) \cdot \omega(G))^r$.

Generalizations. Various generalizations of CIS graphs have been studied in the literature, including general partition graphs, equistable graphs, and triangle graphs. *General partition graphs* were introduced by McAvaney *et al.* [15] as intersection graphs of set systems such that each maximal stable set corresponds to a partition of the ground set; they are exactly the graphs in which every edge is contained in a strong clique. *Equistable graphs* were introduced by Payan [20] in 1980 as graphs G such that there exists a non-negative weight function on the set of vertices for which every maximal stable set has total weight 1, and these are the only vertex-subsets of weight 1. A graph G is a *triangle graph* if for every maximal stable set S and every edge $uv \in E(G - S)$, there exists a vertex $w \in S$ forming a triangle with u and v (see, for example, [14]). The complexity status of recognizing general partition graphs, equistable graphs, or triangle graphs is open, and no combinatorial characterization of equistable graphs is known. In a personal communication in 2009, Jim Orlin proved that every general partition graph is equistable and conjectured that the converse assertion is valid (see [16]). The conjecture was disproved by Milanič and Trotignon [18], who constructed counterexamples among the complements of line graphs of triangle-free graphs, using connections with matchings. These generalizations of the CIS property are related as follows: CIS $\subset$ general partition $\subset$ equistable $\subset$ triangle. The validity of Orlin's conjecture for perfect graphs remains an open question (see [18]).

A different generalization of CIS graphs can be obtained by identifying a graph G with a 2-edge-coloring of the complete graph with vertex-set $V(G)$ obtained by coloring the edges of G red and the edges of its complement blue. For any $d \geq 2$, a d-edge-coloring of a complete graph is said to be *CIS* if every collection of maximal stable sets, one for each spanning subgraph given by all the edges of the same color, has a non-empty intersection (see [11]). In this context, we mention the 'Δ-conjecture' posed by Vladimir Gurvich in 1978 in his Ph.D. thesis. This conjecture states that in every CIS d-edge-coloring of a complete graph, no triangle is colored with three different colors. The conjecture can be equivalently stated as follows: for every edge-coloring of a complete graph such that some triangle is colored with three distinct colors, there is a collection of maximal stable sets, one in each monochromatic spanning subgraph, that have empty intersection. If true, the Δ-conjecture would reduce the study of CIS d-edge-colorings of complete graphs – which have applications in combinatorial game theory – to the study of CIS graphs (see, for example, [2] and [10]).

Further results and references on CIS graphs can be found in [2] and [3], and on related classes in [4] and [17].

References

[1] L. Alcón, M. Gutierrez, and M. Milanič, A characterization of claw-free CIS graphs and new results on the order of CIS graphs. Electronic Notes in Theoretical Computer Science **346** (2019), 15–27.

[2] D. V. Andrade, E. Boros, and V. Gurvich, On graphs whose maximal cliques and stable sets intersect, *Optimization Problems in Graph Theory* (ed. B. Goldengorin), Springer International (2018) 3–63; see also RUTCOR Research Reports (2006), 17-2006.

[3] E. Boros, V. Gurvich, and M. Milanič, On CIS circulants, *Discrete Math.* **318** (2014), 78–95.

[4] E. Boros, V. Gurvich, and M. Milanič, On equistable, split, CIS, and related classes of graphs, *Discrete Appl. Math.* **216** (2017), 47–66.

[5] N. Chiarelli, B. Martínez-Barona, M. Milanič, J. Monnot, and P. Muršič. Strong cliques in diamond-free graphs. *Graph-Theoretic Concepts in Computer Science*, Lecture Notes in Comput. Sci. **12301**, Springer (2020), 261–273.

[6] B. Courcelle, J. A. Makowsky, and U. Rotics, Linear time solvable optimization problems on graphs of bounded clique-width, *Theor. Comput. Syst.* **33** (2000), 125–150.

[7] X. Deng, G. Li, and W. Zang, Proof of Chvátal's conjecture on maximal stable sets and maximal cliques in graphs, *J. Combin. Theory* (B) **91** (2004), 301–325.

[8] E. Dobson, A. Hujdurović, M. Milanič, and G. Verret, Vertex-transitive CIS graphs, *Europ. J. Combin.* **44** (2015), 87–98.

[9] P. A. Grillet, Maximal chains and antichains, *Fund. Math.* **65** (1969), 157–167.

[10] V. Gurvich, Decomposing complete edge-chromatic graphs and hypergraphs. Revisited, *Discrete Appl. Math.* **157** (2009), 3069–3085.

[11] V. Gurvich, On exact blockers and anti-blockers, Δ-conjecture, and related problems, *Discrete Appl. Math.* **159** (2011), 311–321.

[12] A. Hujdurović, Strong cliques in vertex-transitive graphs, *J. Graph Theory* (2020), to appear. DOI: 10.1002/jgt.22573.

[13] A. Hujdurović, M. Milanič, and B. Ries, Graphs vertex-partitionable into strong cliques, *Discrete Math.* **341** (2018), 1392–1405.

[14] T. Kloks, C.-M. Lee, J. Liu, and H. Müller, On the recognition of general partition graphs, *Graph-Theoretic Concepts in Computer Science*, Lecture Notes in Comput. Sci. **2880**, Springer (2003), 273–283.

[15] K. McAvaney, J. Robertson, and D. DeTemple, A characterization and hereditary properties for partition graphs, *Discrete Math.* **113** (1993), 131–142.

[16] Š. Miklavič and M. Milanič, Equistable graphs, general partition graphs, triangle graphs, and graph products, *Discrete Appl. Math.* **159** (2011), 1148–1159.

[17] M. Milanič, Strong cliques and stable sets, to appear in: Topics in Algorithmic Graph Theory, Edited by Lowell W. Beineke, Martin Charles Golumbic, and Robin J. Wilson, Cambridge University Press, 2021.

[18] M. Milanič and N. Trotignon, Equistarable graphs and counterexamples to three conjectures on equistable graphs, *J. Graph Theory* **84** (2017), 536–551.

[19] S.-i. Oum and P. Seymour, Approximating clique-width and branch-width, *J. Combin. Theory* (B) **96** (2006), 514–528.

[20] C. Payan, A class of threshold and domishold graphs: equistable and equidominating graphs, *Discrete Math.* **29** (1980), 47–52.

[21] W. Zang, Generalizations of Grillet's theorem on maximal stable sets and maximal cliques in graphs, *Discrete Math.* **143** (1995), 259–268.

[22] I. Zverovich and I. Zverovich, Bipartite bihypergraphs: a survey and new results, *Discrete Math.* **306** (2006), 801–811.

Crossing numbers of K_n for geometric and topological drawings - A short survey

Oswin Aichholzer

University of Technology Graz, Graz, Austria

In the area of crossing numbers we ask for minimizing the number of edge intersections in a drawing of a graph. There is a rich variety of crossing number problems: Which graphs do we consider, what exactly is a drawing of a graph and on which surface is it drawn, and how are intersections counted? In this talk we will concentrate on the crossing number of complete graphs drawn in the plane, including their rich history around Hill's conjecture.

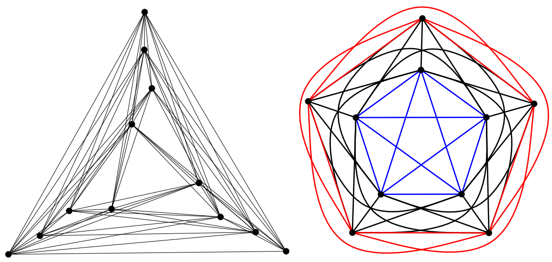

Figure 2: Crossing minimal drawings of K_n. Left: Geometric drawing of K_{12} with 153 crossings. Right: Simple drawing of K_{10} with 60 crossings.

We will have a look at geometric drawings (vertices are points in the plane and edges of the graph are straight line segments), and simple drawings (edges are simple Jordan arcs with at most one pairwise intersection), which are also called simple topological graphs. Our results ar based on a representation of the complete graph with order types (for geometric graphs) and rotation systems (for simple drawings). We will present recent developments, as well as (old and new) open questions.

Keywords: Crossing number, complete gaph, geometric graph, simple drawing

Zeros of modular forms and their derivatives

Sanoli Gun

The Institute of Mathematical Sciences, India

We give an overview of the state-of-the-art on zeros of modular forms and their derivatives. We also report on a recent joint work with Joseph Oesterlé in this set up.

Keywords: Zeros of modular forms, Critical points of Eisenstein series, Chudnovsky's theorem

Eigenvalues and linear programming bound for graphs and hypergraphs

Sebastian Cioabă

University of Delaware, USA

The spectrum of a graph is closely related to many graph parameters. In particular, the spectral gap of a regular graph which is the difference between its valency and second eigenvalue, is widely seen an algebraic measure of connectivity and plays a key role in the theory of expander and Ramanujan graphs. In this paper, I will give an overview of recent work studying the maximum order of a regular graph (bipartite graph or hypergraph) of given valency whose second largest is at most a given value. This problem can be seen as a spectral Moore problem and has close connections to Alon-Boppana theorems for graphs and hypergraphs and with the usual Moore or degree-diameter problem.

Keywords: Eigenvalues, Alon-Boppana theorem, Ramanujan graphs, spectral Moore

Subtrees of trees

Stephan Wagner

Department of Mathematics, Uppsala University, Sweden

Department of Mathematical Sciences, Stellenbosch University, South Africa

By a subtree of a tree T, we mean any nonempty induced subgraph that is connected and thus again a tree. The study of the average number of vertices in a subtree, which is called the *mean subtree order*, goes back to Jamison's work in the 1980s [3,4]. This quantity will be denoted by μ_T, and can be expressed as the total number of vertices in all subtrees of T divided by the number of subtrees. Along with the mean subtree order, Jamison also proposed a normalised version called the *subtree density*. If T has n vertices, then the density is given by

$$D_T = \frac{\mu_T}{n}.$$

Clearly, the density always lies between 0 and 1. In fact, a stronger statement holds: as Jamison proved, paths have the minimum mean subtree order among all trees with n vertices, namely $\frac{n+2}{3}$. Thus the density lies in fact between $\frac{1}{3}$ and 1. The proof that paths indeed attain the minimum is surprisingly intricate.

On the other hand, Jamison showed that the density can be arbitrarily close to 1. While it initially appears that the maximum mean subtree order and subtree density are attained by stars, this fails to be the case as soon as the number of vertices is at least 9. The structure of the trees with maximum mean subtree order is in fact considerably more complicated and has not been fully characterised yet—more about this question later.

A key result that Jamison proved concerns a local version of the mean subtree order. For a subset A of vertices of T, let $\mu_T(A)$ be the average number of vertices in all subtrees that contain A. In particular, $\mu_T(\emptyset) = \mu_T$. The following monotonicity result holds:

Theorem 4 (Jamison [3]) *If $A \subseteq B$, then $\mu_T(A) \leq \mu_T(B)$. Equality holds if and only if the smallest subtree that contains A is also the smallest subtree that contains B.*

In particular, the "local" mean subtree order $\mu_T(v) = \mu_T(\{v\})$ at a vertex v is always greater than the "global" mean subtree order μ_T, a fact that is extremely useful in proving upper bounds.

Jamison left a number of open problems, many of which have only been solved recently or are still open. In the following, some of these problems and their solutions are listed.

Problem 1 (Jamison [3]) *Does a tree with no nodes of degree* 2 *(i.e., a* homeomorphically irreducible tree*) always have density* $\geq \frac{1}{2}$*?*

This question was answered affirmatively by Vince and Wang [6]. In fact, they showed more:

Theorem 5 (Vince and Wang [6]) *For every homeomorphically irreducible tree* T*, we have*

$$\frac{1}{2} \leq D_T < \frac{3}{4}.$$

Haslegrave [2] provided necessary and sufficient conditions for a sequence of homeomorphically irreducible trees whose order tends to infinity to have density tending to $\frac{1}{2}$ or $\frac{3}{4}$. Another question of Jamison's was concerned with whether or not the mean subtree order or density characterises trees of a given order:

Problem 2 (Jamison [3]) *Do nonisomorphic trees of the same order always have different densities?*

However, the answer is negative [7]. In fact, the proportion of trees of order n that are uniquely determined by their densities tends to 0 as n goes to infinity. Two other questions are concerned with the local mean subtree order.

Problem 3 (Jamison [3]) *For any tree* T *and vertex* v *of* T*, is it true that the local mean subtree order at* v *is at most twice the global mean subtree order, i.e.,*

$$\mu_T(v) \leq 2\mu_T?$$

It is clear that it suffices to consider a vertex v where $\mu_T(v)$ is maximal in this problem, which also motivates the second question.

Problem 4 (Jamison [3]) *For any tree T, does the largest local mean subtree order $\mu_T(v)$ always occur at a leaf?*

The answer to Problem 3 is affirmative [8], and the constant 2 is sharp, i.e., cannot be replaced by a smaller constant. However, the largest local mean subtree order does not necessarily occur at a leaf.

Theorem 6 (Wagner and Wang [8]) *The largest local mean always occurs at a leaf or a vertex of degree two. There are infinite families of trees for both cases.*

The following problem of Jamison's, which was settled (affirmatively) by Mol and Oellermann in [5], is motivated by the fact that it leads to a conceptually simpler proof of the fact that the path has the minimum mean subtree order.

Problem 5 (Jamison [3]) *Let u be a neighbour of v that does not lie on the shortest path between v and w. Then the tree T' obtained from T by deleting the edge uv and instead adding uw is called a standard 1-associate of T. Does every tree T that is not a path have a standard 1-associate whose mean subtree order is smaller?*

The only problem in Jamison's first paper [3] that is still open is in fact the first one he stated, and perhaps also the most natural and important of all his questions: what can we say about trees with maximum mean subtree order? Specifically, Jamison put forward the following question.

Problem 6 (Jamison [3]) *Is the tree of maximum density of each order a caterpillar?*

In spite of computational evidence in favour of the "caterpillar conjecture", it has been neither proven nor disproven. A local version of the question was settled very recently, though:

Theorem 7 (Cambie, Wagner and Wang [1]) *The maximum of the local mean subtree order $\mu_T(v)$ among all choices of an n-vertex tree T and a vertex v of the tree is always obtained for a broom, i.e., a tree consisting of a path and leaves attached to one end of the path, where v is the other end of the path.*

This result can also be used to determine a number of characteristics of the trees with maximum mean subtree order.

Theorem 8 (Mol and Oellermann [5]; Cambie, Wagner and Wang [1]) *The maximum of the mean subtree order over all trees with* n *vertices is* $n - 2\log_2 n + O(1)$*. Moreover, trees that attain the maximum have the following properties:*

- *The number of leaves is at most* $4\log_2 n + O(1)$*.*
- *The number of subtrees is* $\Theta(n^4)$*.*
- *The diameter is at least* $n - O(n^{1/2+\delta})$ *for every fixed* $\delta > 0$*.*

References

[1] S. Cambie, S. Wagner, and H. Wang. On the maximum mean subtree order of trees. Preprint, arXiv:2010.15522.

[2] J. Haslegrave. Extremal results on average subtree density of series-reduced trees. *J. Combin. Theory Ser. B*, 107:26–41, 2014.

[3] R. E. Jamison. On the average number of nodes in a subtree of a tree. *J. Combin. Theory Ser. B*, 35(3):207–223, 1983.

[4] R. E. Jamison. Monotonicity of the mean order of subtrees. *J. Combin. Theory Ser. B*, 37(1):70–78, 1984.

[5] L. Mol and O. R. Oellermann. Maximizing the mean subtree order. *J. Graph Theory*, 91(4):326–352, 2019.

[6] A. Vince and H. Wang. The average order of a subtree of a tree. *J. Combin. Theory Ser. B*, 100(2):161–170, 2010.

[7] S. Wagner and H. Wang. Indistinguishable trees and graphs. *Graphs Combin.*, 30(6):1593–1605, 2014.

[8] S. Wagner and H. Wang. On the local and global means of subtree orders. *J. Graph Theory*, 81(2):154–166, 2016.

Parameterizing from triviality: Algorithms on graphs that are close to an easy graph class

Venkatesh Raman

The Institute of Mathematical Sciences, India

Graphs form a central object in discrete mathematics as they model many practical networks including electrical, road and social networks. There are a number of special graph classes studied in literature including trees, bipartite graphs, planar graphs and subclasses of perfect graphs including chordal graphs and interval graphs to name a few [11]. Apart from the fact that most networks that arise in practice seem to belong to one of these classes of graphs, most of these graph classes have the following nice algorithmic properties.

- A graph in these classes can be recognized in time polynomial in the input size.
- Some optimization problems that are NP-hard in general graphs tend to be solvable in polynomial time in these classes of graphs.

For example, Chordal graphs are recognizable in polynomial time, and problems like the MAXIMUM INDEPENDENT SET, MINIMUM VERTEX COVER and CHROMATIC NUMBER are solvable in polynomial time in Chordal graphs. Informally we refer to any of these graph classes as an easy graph class.

In this talk we give an overview of algorithmic problems in graphs that are *close to* one of these easy graph classes, for appropriate measures of distance. For the most part, we look at graphs that have a small number (parameterized as k) of vertices whose deletion places the resulting graph in an easy graph class. We address the recognition problem of these graphs as well as solving some standard optimization problems in these classes of graphs that are otherwise NP-hard in general graphs.

Recognition Problems

Formally let Π be a class of graphs, and let $\Pi + k$ be the class of graphs in which there is a subset of at most k vertices whose deletion results in a graph in Π. Early work on these *graph modification problems* can be traced back to the work of Lewis and Yannanakis [30] who showed that determining whether there are k vertices whose deletion results in a given *hereditary*[†] graph class is NP-complete. This implies that a polynomial time algorithm for the recognition problem of graphs in $\Pi+k$ is unlikely for a hereditary graph class Π. This implies for example, polynomial time algorithms are unlikely to exist for determining whether a given graph has at most k vertices whose deletion results in a bipartite or an edgeless or a chordal or a planar graph as all these graph classes are hereditary.

If we have a polynomial time algorithm to recognize a graph in Π, then recognising graphs on n vertices in $\Pi+k$ can be solved in $n^{k+O(1)}$ by trying all vertex subsets of size k, removing those vertices and seeing whether the resulting graph belongs to Π (by using the polynomial time algorithm for recognition in Π). We address these problems in the paradigm of Parameterized Complexity [8].

In classical complexity, the complexity of a problem is analysed as a function of the input size. In Parameterized Complexity, problems come with some associated parameter k, apart from the input of size n, and the complexity is analysed as a function of both n and k. This allows a finer analysis of the problem complexity, as parameters (beyond the input size) are quite common in practice. In this paradigm, a (parameterized) problem is fixed-parameter tractable (FPT) if it has an algorithm with runtime $f(k)n^c$ for some function of k, where c is a constant independent of k. We also call such an algorithm an FPT algorith. There is also a hierarchy of complexity classes in the paradigm to show evidence of intractability.

For the recognition problem discussed above, the naive $n^{k+O(1)}$ algorithm does not make it fixed-parameter tractable, and so a natural question is whether there is another algorithm that makes the problem fixed-parameter tractable. This was answered in the positive by Cai [4] for graph classes $\Pi+k$ where Π is any hereditary graph that has a finite set of fobidden graphs[‡]. In the case of hereditary graph

[†]A graph class is hereditary if for any graph in the class, all its induced subgraphs also belong to the graph class

[‡]Forbidden set of a hereditary graph class Π is a set F of graphs such that a graph G is not in Π if and only if there is some $H \in F$ that is an induced subgraph of G. It is known that every hereditary graph class can be characterized by a forbidden set which could be infinite.

classes with infinite forbidden sets, tractability results are known when Π is the class of all forests [24], or bipartite graphs [27] or chordal graphs [5,21] or interval graphs [6] to name a few. Intractability results are known when Π is the class of perfect graphs [17] or wheel-free graphs [26].

A dichtomy result characterizing graph classes for which the problem is tractable, and those for which it is intractable is an interesting open problem. Such a dichotomy result is known for the related problem on $\Pi + (n - k)$ graphs, where Π is any hereditary graph class and n is the number of vertices in the graph [25]. This is equivalent to determining whether a given graph on n vertices has $n - k$ vertices whose deletion results in a graph in Π or equivalently whether the given graph has a subset of at least k vertices that induce a subgraph in Π.

Associated with the notion of fixed-parameter tractability, is the notion of kernelizability, where the given instance of a parameterized problem is reduced in polynomial time to an equivalent instance whose size is just a function of the parameter k. See [1,7,10] for more recent work on kernelizability as well as the related edge-modification problems (where one is interested in determining whether a given graph can be placed in a given graph class by adding, deleting or contracting k *edges* of the graph), and on problems on $\Pi + k$ graphs where Π is a non-hereditary graph class.

Recently Jacob et al. [18] have initiated investigation into the problem of deletion to *scattered graph classes*. Here one is given a finite number of graph classes $\Pi_1, \Pi_2, \dots \Pi_c$, and let Π be the class of graphs G where each connected component of G is in some Π_i for some $i = 1$ to c. Then the problem is to recognize graphs in $\Pi + k$ for such Π. I.e. the problem asks whether every *connected component of a given graph* after deletion of k vertices is in some Π_i for some i.

Solving hard problems in $\Pi + k$

In this section, we address the problem of solving some optimization problems that are NP-hard in general graphs, in $\Pi + k$ graphs. Note that Π is a subclass of $\Pi + k$, and so if the problem we want to solve is NP-hard in Π, then there is no hope of an FPT algorithm for it in $\Pi + k$, as the algorithm will be polynomial for a constant k. So let us assume that the problem is polynomial time solvable in Π.

Similarly there is no hope of obtaining an algorithm that is polynomial in n and k for the problem in $\Pi + k$, as any graph G is in $\Pi + k$ for some integer k. Here again, parameterized complexity is a natural paradigm to address the problem, and we ask whether the problem is FPT.

We start with a simple example of the VERTEX COVER problem. Recall that vertex cover in a graph is a subset of vertices whose deletion results in an edge-less graph. In other words, every edge has at least one end point in the vertex cover. The VERTEX COVER problems asks for a minimum sized vertex cover in the graph. While the problem is NP-hard in general graphs, it is trivially solvable in linear time in trees and hence in forests. The algorithm repeatedly picks the neighbor of a leaf vertex in the solution and deletes it and all isolated vertices from the graph. As every tree has a leaf vertex, the process ends up picking the minimum vertex cover in the graph. There is an easy argument for the correctness of the algorithm.

Now consider the problem in the class $\Pi + k$ where Π is the class of forests. This has an $2^k n^{O(1)}$ FPT algorithm as follows. Let S be a subset of vertices of the given graph G such that $G \setminus S$ is a forest.

For $T \subseteq S$

- Find the minimum vertex cover C of $G[V \setminus S \setminus (N(T) \cap (V \setminus S))]$
- $VC(T) = (S \setminus T) \cup C \cup (N(T) \cap (V \setminus S))$.

Output the $VC(T)$ with the smallest cardinality among all $T \subseteq S$.

Here $G[X]$ for a vertex subset X is the induced subgraph of G on X. The algorithm essentially guesses a subset $(S \setminus T)$ of S as the one that intersects with the proposed vertex cover. As T is not in the vertex cover, all its neighbors in $G[V \setminus S]$ must be in the vertex cover. Then the algorithm deletes S and $N(T) \cap (V \setminus S)$ from the graph and finds the minimum vertex cover in the resulting graph. The correctness of the algorithm is easy to see. Clearly the algorithm takes $2^k n^{O(1)}$ as there are 2^k choices for T and $G[V \setminus S \setminus (N(T) \cap (V \setminus S))]$ is a subgraph of $G \setminus S$, and hence is a forest and hence the minimum vertex cover C can be computed in polynomial time, for each T.

Note that the only place where we used the fact that Π is the class of forests is that VERTEX COVER can be solved in polynomial time in the class of forests. For the above algorithm to go through, it is clear that Π can be any class of hereditary graphs where VERTEX COVER is solvable in polynomial time. Thus we have

Theorem 9 *Let Π be a class of hereditary graphs where the* VERTEX COVER *problem can be solved in polynomial time. Then* VERTEX COVER *is fixed-parameter tractable in the class of graphs $\Pi + k$ when parameterized by k.*

As VERTEX COVER is polynomial time solvable in the class of perfect graphs [16], as a corollary we have

Corollary 1 *Vertex Cover is fixed-parameter tractable in the class $\Pi + k$ where Π is the class of perfect graphs.*

Scaling the algorithm for VERTEX COVER to other problems is more involved. See [14,15] for algorithms for the DOMINATING SET problem on graph class $\Pi + k$ where Π is the class of split graphs or cluster graphs where the DOMINATING SET is polynomial time solvable. See [23] for a problem called CYCLE PACKING in $\Pi + k$ graphs where Π is the class of proper interval graphs where the problem is solvable in polynomial time.

When Π is the class of edgeless graphs or the class of forests, graphs in $\Pi + k$ have treewidth bounded by at most $k + 1$ and hence many NP-hard problems have an FPT algorithm on such graphs (See [8] for the notion of treewidth and the algorithms mentioned here).

Note that in Theorem 9, we are looking at two parameters of the graph: vertex cover size, and deletion distance to the graph class Π. In general, this opens up a rich variety of parameterizations where one can investigate the complexity of optimizing one parameter when parameterized by another parameter. This line of investigation [13,20] goes by the name *structural parameterization* or the *parameter ecology program*. See [12,29] for the parameterized complexity of the VERTEX COVER problem in $\Pi + k$ graphs for several graphs classes Π, and see [22,28] for similar results for the FEEDBACK VERTEX SET problem.

Notice that in the proof of Theorem 9, we assumed that the deletion set S is given with the input. This assumption can be removed whenever the deletion set can be found by a FPT algorithm. More recently, Jacob et al. [19] have opened up a new research direction where certain NP-hard problems are solved more efficiently in $\Pi + k$ graphs (when Π is specifically the class of chordal graphs) than actually finding the modulator and then solving the problem. See [9] for algorithms for optimization problems in almost chordal graphs, graphs that can be turned into a chordal graph by adding at most k edges.

Conclusions

To conclude, both problem directions we have surveyed have seen a lot of work recently, and we have only scratched the surface. There are still a number of open problems. For example,

- while it is known that recognising graphs in $\Pi + k$ where Π is the class of Perfect graphs, is intractable in the parameterized complexity framework, can optimization problems (that are polynomial time solvable in Perfect Graphs) be solved in FPT time in this class of graphs without finding the modulator along the lines of the work of Jacob et al. [19]?

- Can the 2^k bound be improved for the $f(k)$ function of the FPT algorithm of Theorem 1? For the problem in Theorem 1, Bodlaender et al. [2] have shown that there is no polynomial kernel (under complexity theoretic assumptions) if Π is the class of CLIQUES. As this class is a subclass of the class of all Chordal or Interval or Perfect Graphs, a similar result follows for these classes of graphs too. See [3] for a recent work on kernelization for VERTEX COVER on $\Pi + k$ graphs for some restricted (more precisely, minor-closed) families Π.

The reader is encouraged to refer to the surveys and papers listed for more recent results and open problems.

References

[1] Akanksha Agrawal, Graph Modification Problems beyond the known boundaries, PhD thesis, University of Bergen, Norway,

[2] Hans L. Bodlaender, Bart M. P. Jansen, Stefan Kratsch, Kernelization Lower Bounds by Cross-Composition. SIAM J. Discret. Math. **28(1)**: 277-305 (2014).

[3] Marin Bougeret, Bart M. P. Jansen and Ignasi Sau, Bridge-Depth Characterizes Which Structural Parameterizations of Vertex Cover Admit a Polynomial Kernel, *in Proceedings of 47th ICALP* 16:1-16:19, 2020.

[4] L. Cai, Fixed-parameter tractability of graph modification problems for hereditary properties, *Information Processing Letters* **58(4)** 171–176 1996.

[5] Yixin Cao and Dániel Marx. Chordal editing is fixed-parameter tractable. *Algorithmica* **75(1)** 118–137 2016.

[6] Yixin Cao and Dániel Marx. Interval deletion is fixed-parameter tractable. *ACM Transactions on Algorithms* **11 (3)** 2015.

[7] Christophe Crespelle, Pål Grønås Drange, Fedor V. Fomin, Petr A. Golovach, A survey of parameterized algorithms and the complexity of edge modification, *arXiv:2001.06867* Feb 2020.

[8] M. Cygan, F. V. Fomin, L. Kowalik, D. Lokshtanov, D. Marx, M. Pilipczuk, M. Pilipczuk, and S. Saurabh. Parameterized Algorithms, Springer, 2015.

[9] Fedor V. Fomin, Petr A. Golovach, Subexponential parameterized algorithms and kernelization on almost chordal graphs, arXiv:2002.08226, preliminary version in *the proceedings of ESA 2020*.

[10] Fedor V. Fomin, Saket Saurabh and Neeldhara Misra, Graph Modification Problems: A Modern Perspective, in Wang J., Yap C. (eds) Frontiers in Algorithmics (FAW) Lecture Notes in Computer Science, **vol 9130** Springer 2015.

[11] M.C. Golumbic, Algorithmic Graph Theory and Perfect Graphs, Annals of Discrete Mathematics, **vol. 57** 2nd edn. Elsevier, 2004.

[12] Michael R. Fellows, Bart M.P. Jansen, and Frances Rosamond. Towards fully multivariate algorithmics: Parameter ecology and the deconstruction of computational complexity. *European Journal of Combinatorics* **34(3)** :541–566, 2013.

[13] Michael R. Fellows, Daniel Lokshtanov, Neeldhara Misra, Matthias Mnich, Frances A. Rosamond, Saket Saurabh, The Complexity Ecology of Parameters: An Illustration Using Bounded Max Leaf Number, *Theory of Computing Systems* **45(4)** 822-848 2009

[14] Dishant Goyal, Ashwin Jacob, Kaushtubh Kumar, Diptapriyo Majumdar and Venkatesh Raman, Structural Parameterizations of Dominating Set Variants, *International Computer Science Symposium in Russia (CSR) Computer Science – Theory and Applications* 157-168, 2018.

[15] Neeldhara Misra and Piyush Rathi, The Parameterized Complexity of Dominating Set and Friends Revisited for Structured Graphs, *International Computer Science Symposium in Russia (CSR) Computer Science – Theory and Applications* 299-310, 2019.

[16] Martin Grötschel, László Lovász, Alexander Schrijver, Geometric Algorithms and Combinatorial Optimization, Springer-Verlag (1988) 273–303.

[17] Pinar Heggernes, Pim van 't Hof, Bart M. P. Jansen, Stefan Kratsch, and Yngve Villanger. Parameterized complexity of vertex deletion into perfect graph classes. Theoretical Computer Science, **511** 172–180, 2013.

[18] Ashwin Jacob, Diptapriyo Majumdar and Venkatesh Raman, Parameterized complexity of deletion to scattered graph classes, to appear in *the Proceedings of International Symposium on Parameterized and Exact Computation (IPEC)* 2020.

[19] Ashwin Jacob, Fahad Panolan, Venkatesh Raman and Vibha Sahlot, Structural Parameterization with Modulator Oblivion, to appear in *the Proceedings of International Symposium on Parameterized and Exact Computation (IPEC)* 2020.

[20] B. M. P. Jansen, *The Power of Data Reduction: Kernels for Fundamental Graph Problems*, PhD thesis, Utrecht University, The Netherlands, 2013.

[21] Bart M.P. Jansen and Marcin Pilipczuk, Approximation and kernelization for chordal vertex deletion, In *Proceedings of the Twenty-Eighth Annual ACM-SIAM Symposium on Discrete Algorithms* pages 1399–1418. Society for Industrial and Applied Mathematics, 2017.

[22] Bart M.P. Jansen, Venkatesh Raman, and Martin Vatshelle, Parameter ecology for feedback vertex set, Tsinghua Science and Technology, **19(4)** 387–409, 2014.

[23] R. Krithika, Abhishek Sahu, Saket Saurabh, Meirav Zehavi, The Parameterized Complexity of Cycle Packing: Indifference is Not an Issue. *Algorithmica* **81(9)** 3803-3841, 2019.

[24] Tomasz Kociumaka and Marcin Pilipczuk, Faster deterministic Feedback Vertex Set, *Information Processing Letters*, **114(10)**: 556–560, 2014.

[25] S. Khot and V. Raman, Parameterized Complexity of finding subgraphs with hereditary properties, *Theoretical Computer Science* **289** 997-1008, 2002.

[26] Daniel Lokshtanov. Wheel-free deletion is W[2]-hard, In *Parameterized and Exact Computation, Third International Workshop, IWPEC 2008* Victoria, Canada, May 14-16, 2008. pages 141–147, 2008.

[27] Daniel Lokshtanov, N. S. Narayanaswamy, Venkatesh Raman, M. S. Ramanujan, Saket Saurabh, Faster Parameterized Algorithms Using Linear Programming. ACM Transactions on Algorithms **11(2)** 15:1-15:31, 2014.

[28] Diptapriyo Majumdar and Venkatesh Raman, Structural parameterizations of undirected feedback vertex set: FPT algorithms and kernelization, *Algorithmica* **80(9)** 2683–2724, 2018.

[29] Diptapriyo Majumdar, Venkatesh Raman, and Saket Saurabh, Polynomial kernels for vertex cover parameterized by small degree modulators, *Theory of Computing Systems*, **62(8)** 1910–1951, 2018.

[30] Lewis, J.M., Yannakakis, M., The node-deletion problem for hereditary properties is np-complete, *Journal of Computer and System Sciences* **20(2)**, 219–230, 1980.

Nowhere-zero flows in signed planar graphs

Jiaao Li[1], Yulai Ma[2], Zhengke Miao[3], Yongtang Shi[2] and Weifan Wang[4]

[1]School of Mathematical Sciences and LPMC, Nankai University, China

[2]Center for Combinatorics and LPMC, Nankai University, China

[3]Research Institute of Mathematical Science and School of Mathematics and Statistics, Jiangsu Normal University, China

[4]Department of Mathematics, Zhejiang Normal University, China

Besides the Four Color Theorem, one of the most influential planar graph coloring results is the Grötzsch's theorem that every triangle-free planar graph is vertex 3-colorable. As the dual of graph coloring, Tutte [10,11] initiated the study of flow theory and observed that a plane graph admits a nowhere-zero k-flow if and only if its dual graph is vertex k-colorable. Here, a nowhere-zero k-flow (k-NZF for short) of a graph G is an orientation together with an edge-mapping $f : E(G) \mapsto \{\pm 1, \pm(2), \ldots, \pm(k-1)\}$ such that the total incoming flow equals the total outgoing flow at each vertex. Motivated by the Grötzsch's theorem, Tutte proposed the following 3-Flow Conjecture.

Conjecture. *Every 4-edge-connected graph admits a nowhere-zero 3-flow.*

As a major open problem in the flow theory, Tutte's 3-flow conjecture is still open as of today but many significant progresses have been made [3,5,7,9], and the readers are referred to [4] for a recent survey on this topic.

The concept of flows are naturally generalized to signed graphs, motivated from the study of graphs embedding on non-orientable surfaces, where nowhere-zero flows emerge as the dual notion to local tensions. Flows of signed graphs are defined similarly, except the orientation of each negative edge is directed as both away from or both towards its ends. It is natural to consider the analogy of Tutte's 3-flow conjecture for signed graphs. However, this fails in general as to be explain below. Youngs [13] constructed infinite families of triangle-free projective plane graphs with chromatic number 4, and he also proved that a quadrangulation of projective plane graph has chromatic number either 2 or 4 but not 3. Thus by Bouchet's flow-coloring duality theorem [1], there exist infinitely many 4-edge-connected signed projective plane graphs admitting no 3-NZF. Moreover, Máčajová and Škoviera [6] showed that a signed eulerian graph admits a 3-NZF if and only if it can be edge-decomposed into three eulerian subgraphs sharing a common vertex and each of which contains odd number of negative edges. This particularly implies the following proposition.

Proposition 1. *Every* 4*-regular signed graph with odd number of negative edges does not admit a nowhere-zero* 3*-flow. In particular, there exist infinitely many* 4*-edge-connected signed planar graphs with three negative edges that do not admit nowhere-zero* 3*-flows, but they admit nowhere-zero* 4*-flows.*

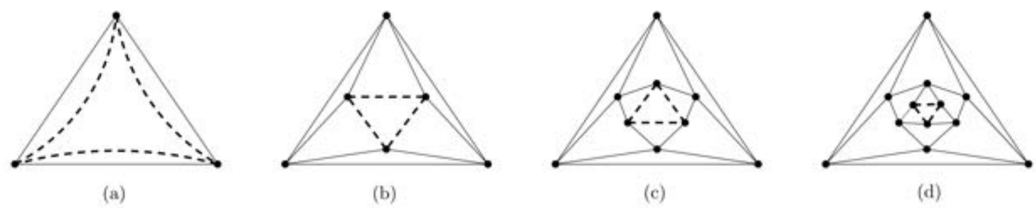

Figure 3: Some 4-edge-connected planar signed graphs without 3-NZF.

An infinite family of 4-edge-connected planar signed graphs with three negative edges that do not admit 3-NZFs is shown in Figure 3. This leaves a possible case on signed graphs with two negative edges (as each signed graph with exactly one negative edge is not flow-admissible). We observe that this case is indeed equivalent to Tutte's 3-Flow Conjecture for ordinary graphs.

Proposition 2. *The following are equivalent.*

(a)(Tutte's 3-flow conjecture) *Every* 4*-edge-connected graph admits a nowhere-zero* 3*-flow.*

(b)(Signed Version with two negative edges) *Every* 4*-edge-connected signed graph with two negative edges admits a nowhere-zero* 3*-flow.*

Motivated by the above facts on 3-NZFs and some recent developments of flows in signed graphs [2, 12], the purpose of this paper is to prove the planar version of Proposition 2(b) (without using Tutte's 3-flow conjecture), which, in some sense, generalizes Grötzsch's theorem and supports Tutte's 3-Flow Conjecture.

Main Theorem. *Every* 4*-edge-connected signed planar graph with two negative edges admits a nowhere-zero* 3*-flow.*

The proof of Main Theorem employs some flow extension ideas from Steinberg and Younger [7] in their proof of flow version Grötzsch's theorem and from Thomassen [8,9] in his proofs of Grötzsch's theorem and the weak 3-flow conjecture. Moreover, due to the existence of negative edges, we need not only to handle the small cuts in a potential counterexample but also to treat the negative edges in various location for different cases. After some prior reductions on the structure of potential counterexamples, we apply some discharging arguments to find reducible configurations in several different situations to complete the proof.

Keywords: Integer flow, Tutte's 3-flow conjecture, Grötzsch theorem, Signed graph

References

[1] A. Bouchet, Nowhere-zero integral flows on a bidirected graph, J. Combin. Theory Ser. B, 34 (1983), 279–292.

[2] M. DeVos, J. Li, Y. Lu, R. Lu, C.-Q. Zhang, Z. Zhang, Flows on flow-admissible signed graphs, J. Combin. Theory Ser. B, 2020, online available: https://doi.org/10.1016/j.jctb.2020.04.008

[3] F. Jaeger, Flows and generalized coloring theorems in graphs, J. Combin. Theory Ser. B, 26 (1979), 205–216.

[4] H.-J. Lai, R. Luo, C.-Q. Zhang, Integer flow and orientation, in: L. Beineke, R. Wilson (Eds.), Topics in Chromatic Graph Theory, in: Encyclopedia of Mathematics and its Applications, vol. 156, 2015, pp. 181–198.

[5] L.M. Lovász, C. Thomassen, Y. Wu, C.-Q. Zhang, Nowhere-zero 3-flows and modulo k-orientations, J. Combin. Theory Ser. B, 103 (2013), 587–598.

[6] E. Máčajová, M. Škoviera, Nowhere-zero flows on signed eulerian graphs, SIAM J. Discrete Math., 31 (2017), 1937–1952.

[7] R. Steinberg, D.H. Younger, Grötzsch theorem for the projective plane graph, Ars Combin., 28 (1989), 15–31.

[8] C. Thomassen, Grötzsch 3-color theorem and its counterparts for the torus and the projective plane graph, J. Combin. Theory Ser. B, 62 (1994), 268–279.

[9] C. Thomassen, The weak 3-flow conjecture and the weak circular flow conjecture, J. Combin. Theory Ser. B, 102 (2012), 521–529.

[10] W. T. Tutte, On the imbedding of linear graphs in surfaces, Proc. London Math. Soc., Ser. 2, 51 (1949), 474–483.

[11] W. T. Tutte, A contribution to the theory of chromatical polynomials, Canad. J. Math., 6 (1954), 80–91.

[12] Y. Wu, D. Ye, W. Zang, C.-Q. Zhang, Nowhere-zero 3-flow in signed graphs, SIAM J. Discrete Math., 28(3) (2014), 1628–1637.

[13] D. A. Youngs, 4-chromatic projective graphs, J. Graph Theory, 21 (1996), 219–227.

Contributed Talks

C04

Cordialness of corona product of star and path

V. Murugan[1] and K. Prabakaran[2]

[1]VIT University, Vellore, Tamil Nadu, India

[2]SRM Institute of Science and Technology, Chennai, Tamil Nadu, India

The corona $G \odot H$ of two graphs G with $V(G) = \{v_1, \ldots, v_{p_1}\}$ and H with p_2 vertices is a graph obtained by taking one copy of G and p_1 copies of H , and then for each $v_i \in V(G)$, joining the vertex v_i with an edge to every vertex in the i^{th} copy of H. In this paper we prove that the corona product of Star and Path admits a cordial labelling.

C08

On high-girth expander graphs with localized eigenvectors

Shohei Satake

Kumamoto University, Japan

One of the main aims of spectral graph theory is to study properties or structures of graphs using spectrum of their adjacency matrices. For regular graphs, Brooks and Lindenstrauss proved an upper bound on their girth (i.e. the length of the shortest contained cycles) via localization of eigenvectors; later this bound was improved by Ganguly and Srivastava and they also proved the sharpness of their bound using probabilistic methods. On the other hand, Alon, Ganguly and Srivastava (to appear in Israel J. Math.) gave an *explicit* construction of high-girth $(d+1)$-regular graphs for a prime d which are optimal (up to some constants) with respect to the bound due to Ganguly and Srivastava. Remarkably, their constructed graphs are near-Ramanujan graphs (i.e. having near-optimal spectral gaps). However, there is no known explicit constructions of optimal high-girth $(d+1)$-regular (near-Ramanujan) graphs for non-prime d.

In this talk, we extend the explicit construction by Alon, Ganguly and Srivastava to general degrees. Specifically, we first present an explicit construction of high-girth $(d+1)$-regular graphs from known Ramanujan graphs, which are optimal (up to some constants) with respect to the bound due to Ganguly and Srivastava for every

$d \geq 29$. Moreover our construction provides near-Ramanujan graphs for almost all degrees as well. These results are based on the idea from Cioabă and Murty, together with some known results on gaps between primes.

On some combinatorial interpretations of Rogers–Ramanujan type identities C12

V.Gupta and M.Rana

School of Mathematics, Thapar Institute of Engineering and Technology, India

In this paper we provide combinatorial interpretations of some Rogers–Ramanujan type identities listed in Chu-Zhang and Slater's Compendium. The tools included, to provide the combinatorial interpretations, are Split part n–color partition, n–color overpartitions and 2–color F–partitions.

Vertex degree in the power graph of a finite abelian group C13

Shubh N. Singh[1] and Amit Sehgal[2]

[1]Central University of South Bihar, India

[2] Pt. NRS Govt. College, Haryana, India

Kelarev and Quinn [1] introduced the *directed power graph* of a semigroup S as a digraph whose vertex set is S itself, and there is an edge from vertex u to other vertex v whenever v is a power of u. Later, Chakrabarty et al. [5] defined the *power graph* $\mathcal{G}(S)$ of a semigroup S as a graph with S as its vertex set, and there is an edge between any two distinct vertices if one is a power of the other.

Chakrabarty et al. [5] proved that the power graph $\mathcal{G}(G)$ of a finite group G is complete if and only if G is a cyclic group of prime-power order. Cameron and Ghosh [7] proved that abelian groups with isomorphic power graphs are isomorphic. Cameron [6] proved that groups with isomorphic power graphs have the same numbers of elements of each order. Curtin and Pourgholi [4] proved that, among all groups of a given order, the power graph of cyclic group of that order has the maximum number of edges. Doostabadi et al. [2] gave a formula for counting the vertex degree in the power graph of cyclic group.

In this paper, we derive a formula for vertex degree in the power graph of a prime-power abelian group. We finally give a general formula for vertex degree in the power graph of a finite abelian group.

References

[1] A. V. Kelarev and S. J. Quinn. Directed graphs and combinatorial properties of semigroups. *Journal of Algebra*, 251(1): 16–26, 2002.

[2] A. Doostabadi, A. Erfanian, and A. Jafarzadeh. Some results on the power graphs of finite groups. *ScienceAsia*, 41: 73–78, 2015.

[3] A. R. Moghaddamfar, S. Rahbariyan and W. J. Shi. Certain properties of the power graph associated with a finite group. *Journal of Algebra and Its Applications*, 13(7): 1450040, 2014.

[4] B. Curtin and G. R. Pourgholi. Edge-maximality of power graphs of finite cyclic groups. *Journal of Algebraic Combinatorics*, 40(2): 313–330, 2014.

[5] I. Chakrabarty, S. Ghosh, and M. K. Sen. Undirected power graphs of semigroups. *Semigroup Forum*, 78(1): 410–426, 2009.

[6] P. J. Cameron. The power graph of a finite group, II. *Journal of Group Theory*, 13(6): 779–783, 2010.

[7] P. J. Cameron and S. Ghosh. The power graph of a finite group. *Discrete Mathematics*, 311(13): 1220–1222, 2011.

Corona operator on Italian domination C14

Jismy Varghese[1] and Aparna Lakshmanan S.[2]

[1]School of Computer Science, DePaul Institute of Science and Technology, Kerala, India

[2]Department of Mathematics, St.Xavier's College for Women, Kerala, India

An Italian dominating function (IDF), of a graph G is a function $f : V(G) \to \{0, 1, 2\}$ satisfying the condition that for every $v \in V(G)$ with $f(v) = 0, \sum_{u \in N(v)} f(u) \geq 2$. The weight of an IDF on G is the sum $f(V) = \sum_{v \in V(G)} f(v)$ and Italian domination number, $\gamma_I(G)$ is the minimum weight of an IDF. In this paper, we study the impact of corona operator and addition of twins on Italian domination number.

Co-Secure domination number of generalized Sierpinski graphs C17

Gisha Saraswathy[1], Manju K Menon[2] and Satheesh E N[1]

[1] NSS Hindu College, Changanacherry, India

[2]St.Paul's College, Kalamassery, India

Let $G = (V, E)$ be a graph. $S \subseteq V$ is a dominating set of G if every vertex in $V \setminus S$ is adjacent to a vertex in S. A dominating set S is called co-secure dominating set if for each $u \in S$ there exist $v \in V \setminus S$ such that v is adjacent to u and $S_1 = (S - \{u\}) \cup \{v\}$ is a dominating set. The minimum cardinality of a co-secure dominating set in G is the co-secure domination number $\gamma_{cs}(G)$ of G and the corresponding set is called γ_{cs}-set.

Generalized Sierpinski graph of dimension t denoted by $S(G, t)$ is the graph with vertex set V^t and edge set defined as follows: $\{u, v\}$ is an edge if and only if there exist $i \in \{1, 2, \cdots, t\}$ such that

1. $u_j = v_j$, if $j < i$
2. $u_i \neq v_i$ and $\{u_i, v_i\} \in E$
3. $u_j = v_i$ and $v_j = u_i$ if $j > i$

where V^t is the set of words of size t on alphabet V. The letters of a word u of length t are denoted by $u_1 u_2 \cdots u_t$.

In this paper, we have obtained the co-secure domination number of generalized sierpinski graph of paths, cycles, and complete graphs.

Keywords: Co-Secure Domination number, Sierpinski Graph, Generalized Sierpinski Graphs

---

C18

Pendant domination in grid graphs

Purushothama S[1], Puttaswamy[2] and Nayaka S[2]

[1]Department of Mathematics, Maharaja Institute of Technology Mysore, Karanataka, India

[2]Department of Mathematics, P.E.S. College of Engineering, Mandya, Karanataka, India

Let $G(V, E)$ be a nontrivial, simple, finite and undirected graph. A subset S of vertices is called a dominating set of G if every vertex not in S is adjacent to at least one vertex in S. The minimum cardinality of a dominating set is called the domination number, denoted by $\gamma(G)$. A dominating set S in G is called a pendant dominating set if induced subgraph of S contains at least one pendant vertex. The minimum cardinality of a pendant dominating set is called the pendant domination number, denoted by $\gamma_{pe}(G)$. In this paper, we determine the pendant domination number of some grid graphs.

Bounds on the Steiner Wiener index of graphs C20

Rasila V. A. and Ambat Vijayakumar

Cochin University of Science and Technology, India

Let S be a set of vertices of a connected graph G. The Steiner distance of S is the minimum size among all connected subgraphs of G containing the vertices of S. The sum of all Steiner distances on sets of size k is called the Steiner k-Wiener index. We obtain formulae for bounds on the Steiner Wiener index of graphs in terms of chromatic number, vertex connectivity and independence number. Also, we obtain bounds for 4-cycle free graphs.

Some energy bounds for complex unit gain graphs in terms of the vertex cover number C22

Aniruddha Samanta and M. Rajesh Kannan

Department of Mathematics, Indian Institute of Technology Kharagpur, India

A $\mathbb{T}$-gain graph, $\Phi = (G, \varphi)$, is a graph in which the function φ assigns a unit complex number to each orientation of an edge of G, and its inverse is assigned to the opposite orientation. The associated adjacency matrix $A(\Phi)$ is defined canonically. The energy $\mathcal{E}(\Phi)$ of a $\mathbb{T}$-gain graph Φ is the sum of the absolute values of all eigenvalues of $A(\Phi)$. We study the notion of vertex energy of a $\mathbb{T}$-gain graph, and establish bounds for it. For any $\mathbb{T}$-gain graph Φ, we prove that $2\tau(G) - 2c(G) \leq \mathcal{E}(\Phi) \leq 2\tau(G)\sqrt{\Delta(G)}$, where $\tau(G), c(G)$ and $\Delta(G)$ are the vertex cover number, the number of odd cycles and the largest vertex degree of G, respectively. Furthermore, using the properties of vertex energy, we characterize the classes of $\mathbb{T}$-gain graphs for which $\mathcal{E}(\Phi) = 2\tau(G) - 2c(G)$ holds. Also, we characterize the classes of $\mathbb{T}$-gain graphs for which $\mathcal{E}(\Phi) = 2\tau(G)\sqrt{\Delta(G)}$ holds. This characterization solves a general version of an open problem.

C24
An approximate functional equation for elements of the Selberg class

Usha K. Sangale[1] and S. B. Nimse[2]

[1]SRTM University, Maharashtra, India

[2]University of Lucknow, India

Let $F(s) = \sum_{n=1}^{\infty} a(n)/n^s,\ s = \sigma + it$ be an L-function, convergent in say, $\sigma > \sigma_0$. In many situations, in the region $\sigma \leq \sigma_0$, it is desirable to express the function $F(s)$ as a finite Dirichlet series with a good error term. Such expressions are popularly known as *approximate formulas* or *approximate functional equation for $F(s)$*. In this talk, we shall derive an approximate functional equation for elements of the Selberg class.

C30
On λ-design conjecture

Ajeet Kumar Yadav

Department of Mathematics, University of Mumbai, India

Let v and λ be integers with $0 < \lambda < v$. A λ-design D is a pair (X, β), where X is a finite set with v elements called points and β is a family of subsets of X called blocks, with $|\beta| = |X|$ such that

1. for all $B_i, B_j \in \beta$, $i \neq j$, $|B_i \cap B_j| = \lambda$;

2. for all $B_j \in \beta$, $|B_j| = k_j > \lambda$, and not all k_j are equal.

The only known examples of λ-designs are so called type-1 designs, which are obtained from symmetric designs by a certain complementation procedure. Ryser and Woodall had independently conjectured that all λ-designs are type-1. Suppose r and $r^*(r > r^*)$ are replication numbers of D and let E and E^* denote the points of X with replication numbers r and r^* respectively. Let $g = \gcd(r-1, r^*-1), m = \gcd((r-r^*)/g, \lambda)$. For distinct points x and y of D, let $\lambda(x, y)$ denote the number of blocks of X containing x and y.

It was obtained by Yadav, Pawale and Shrikhande, that there exists integer $t(x,y)$ such that

$$\lambda(x,y) = \begin{cases} \dfrac{r(r-1)}{(v-1)} & + \quad \dfrac{(r-r^*)}{(v-1)m'}t(x,y) \quad \text{for} \quad r(x)=r(y)=r, \\ \dfrac{r^*(r^*-1)}{(v-1)} & + \quad \dfrac{(r-r^*)}{(v-1)m'}t(x,y) \quad \text{for} \quad r(x)=r(y)=r^*, \\ \dfrac{r(r^*-1)}{(v-1)} & + \quad \dfrac{(r-r^*)}{(v-1)m'}t(x,y) \quad \text{for} \quad r(x)\neq r(y), \end{cases}$$

where $m' = m/2$ if m is even, otherwise $m' = m$.

Further, it was observed that if $|t(x,y) - t(x,y')| \leq m'$, for $x \in E$ and distinct y, y' in $E \setminus \{x\}$ or E^*, then D is type-1. In this paper we investigate the possibilities of Ryser designs to be of type-1 under the condition that $|t(x,y) - t(x,y')| \leq 2m'$. Under this condition, we prove that if $\rho \leq 3$, then Ryser design D is of type-1. Using this we prove that if $\lambda \leq 3$, then D is of type-1. Also, we obtained the exceptions on λ-designs with two block sizes on v points with $v = np+1$, where p is a positive prime and $n \in \{11, 13, 14, \ldots, 22\}$.

One-to-one conditional path covers on augmented cubes C31

S. A. Kandekar, S. A. Mane and B. N. Waphare

Center for Advanced Studies in Mathematics, Department of Mathematics, Savitribai Phule Pune University, India.

It is well known that the hypercube Q_n is one of the most popular interconnection networks because of its noteworthy properties such as maximum connectivity and effective routing algorithms. The augmented cube AQ_n, is proposed by Choudam and Sunitha as an improvement over the hypercube Q_n, which owns more desirable properties than the hypercube, besides keeping some beneficial properties of Q_n. This paper deals with the well-established problem of handling the maximum possible number of one-to-one communication requests without using a single node more than once and keeping required pair/ pairs of vertices on different paths in augmented cubes.

A major goal in the design of a system is to improve efficiency and fault-tolerant capacity. These systems are made fault-tolerant and efficient by providing redundant or spare processors. By keeping two or more active or non compatible processors on different paths, we can make these system more efficient. Thus, we strengthen the parallel path result.

Thus, the problem under study is to find the values of l and k such that for any $(l+2)$ vertices u, v and $w_1, w_2, \ldots, w_l$ with each $w_i \notin \{u^c, v^c\}$, there exists t-path cover in between u and v for $2 \leq t \leq k$ in which w_i and w_i^c lie on different path.

In this paper, we solve the above problem for $l = 1$, $k = 2n-1$ and $l = (2^n - 4)/2$, $k = 2$. i.e. we show that for $2 \leq t \leq (2n-1)$ and given any three distinct vertices u, v and w of AQ_n, there exists t disjoint path cover (t-DPC) in between u and v such that the given vertex w and its complement always lie on different paths. Also for given any two vertices u and v, there exists 2 disjoint path cover (2-DPC) in between u and v such that for every pair w and w^c other than $\{u, u^c, v, v^c\}$, w and w^c lie on different paths. In the first result we obtained, the value of k is optimal and in the second result, the value of l is optimal. Thus, to increase the efficiency and fault-tolerant capacity of the system, the requirement of two processors to be always on two different paths is fulfilled by the result proved in this paper.

C38 On the distance spectra and energy of some graph classes

Anu Varghese

Bishop Chulaparambil Memorial College, Kottayam, India

The distance matrix, distance eigenvalue and distance energy of a connected graph have been studied intensively in the literature. We propose a new problem of studying how the distance energy changes when an edge is deleted. In this paper, we prove that the distance energy of a complete bipartite graph is always increased when an edge is deleted. Also, we compare the distance spectra and energy of graphs to that of its induced subgraphs. We obtain a new characterization for distance hereditary graphs using distance spectrum.

On uniform number of graphs and other graph invariants C40

Elakkiya Mohankumar[1] and Kumar Abhishek[2]

[1]Department of Mathematics, SRM University, Chennai, India

[2]Department of Mathematics, Amrita School of Engineering Coimbatore, India

In 2020, Elakiyya et al., introduced the notion of uniform number of a connected graph G, denoted as $\varsigma(G)$, as the least cardinality of a nonempty subset M of the vertex set of G for which the function $f_M : M^c \to \mathcal{P}(X) - \{\emptyset\}$ defined as $f_M(x) = \{D(x,y) : y \in M\}$ is a constant function, where $D(x,y)$ is the detour distance between x and y in G and $\mathcal{P}$ is the power set of $X = \{D(x_i,x_j) : x_i \neq x_j\}$. In the present article, we investigate the behaviour of newly introduced graph parameter uniform number of a graph $\varsigma(G)$ with the domination number $\gamma(G)$, clique number $\omega(G)$, independence number $\beta(G)$ and chromatic number $\chi(G)$ of a graph.

Divisibility and distribution of ℓ-regular overpartitions C41

Chiranjit Ray

Harish-Chandra Research Institute, India

A partition of a positive integer n is defined as a non-increasing sequence of positive integers whose sum is n. Corteel and Lovejoy introduced the notion of overpartition as the number of partition of n with first occurrence of a number may be overlined. In 2003, Lovejoy investigated the ℓ-regular overpartition $\overline{A}_\ell(n)$, which counts the number of overpartitions of n into parts not divisible by ℓ. In this talk, we will discuss the arithmetic density of the set $\left\{0 \le n < X : \overline{A}_{2\ell}(n) \equiv 0 \pmod{2^k}\right\}$ is exactly 1, where k is any positive integer. We will also study the arithmetic properties of Fourier coefficients of certain integer weight modular form and exhibit infinite families of congruences for $\overline{A}_{2\ell}(n)$.

Keywords: Partition; ℓ-Regular overpartition; Eta-quotients; Modular forms; Hecke eigenform; Arithmetic density

---

C43 **Some properties of weighted hexagonal picture automata and weighted logics**

Jaya Abraham[1] and Dersanambika K. S.[2]

[1]Department of Mathematics, Rajagiri School of Engineering and Technology, India

[2]Department of Mathematics, Fatima Mata National College, Kollam, India

In this paper we extended the concept of EMSO logic on rectangular pictures to that on hexagonal pictures. We introduced the concept of existential MSO (EMSO) logic and weighted three dimensional online tessellation automata (W3OTA) for hexagonal pictures.

Also extended various results in the case of picture series to that of hexagonal picture series. In this paper we tried an extended proof of two main results which states that 'definable picture series are recognizable and recognizable picture series are definable' in the case of hexagonal picture series.

C44 **Herscovici's conjecture on product of shadow graph of paths**

S. Saratha Nellainayaki[1] and A. Lourdusamy[2]

[1]Vyasa Arts and Science Women's College, Manonmaniam Sundaranar University, Tamil Nadu, India

[2]Department of Mathematics, St. Xavier's College (Autonomous), Tamilnadu, India

Given a connected graph G, a distribution on a graph G is an assignment of pebbles on the vertices of the graph G. A pebbling move is defined as for any given distribution of pebbles on the vertices of a connected graph G, a removal of two pebbles from some vertex and the placement of one of those pebbles on an adjacent vertex. The t-pebbling number $f_t(G)$ of a simple connected graph G is the smallest positive integer such that for every distribution of $f_t(G)$ pebbles on the vertices of G, we can move t pebbles to any target vertex by a sequence of pebbling moves. Graham conjectured that for any connected graphs G and H, $f(G \times H) \leq f(G)f(H)$. Herscovici further conjectured that $f_{st}(G \times H) \leq f_s(G)f_t(H)$, for any positive integers s and t. In this paper we show that Herscovici's conjecture is true

when G is a shadow graph of path and H is a graph satisfying the $2t$- pebbling property.

Enumeration of cyclic automorphic hypergraphs C45

Kumi Kobata[1] and Yasuo Ohno[2]

[1]Faculty of Engineering, Kindai University, Japan

[2]Mathematical Institute, Tohoku University, Japan

The aim of this talk is to present a counting formula for cyclic automorphic hypergraphs. These graphs are generalizations of ordinary self-complementary graphs to edge colored hypergraphs. The result can be understood as a generalization of a theorem proved by A. Nakamoto, T. Shirakura and S. Tazawa, which was originally called Royle's conjecture.

Some eigenvalue properties of uniform hyperstar using recurrence relation C46

Deepthi Chandran R. and P. B. Ramkumar

Rajagiri School of Engineering and Technology, Kerala, India

Hypergraph is the best tool to represent multiple relationship. Complex form of social, biological, chemical relation can be represented by hypergraphs. Consider a hypergraph H with vertex set $V = \{v_1, v_2, \cdots, v_n\}$ and hyperedge set $E = \{e_1, e_2, \cdots, e_m\}$. Two edges are adjacent if their intersection is non-empty. A neighbourhood of a vertex v_i,denoted by $N(v_i)$ is defined as the collection of vertices in adjacent edges of v_i. A special type of hypergraph, 3-uniform hyperstar is considered here. A detailed study of characteristic equation is made using adjacency matrix of some of the 3-uniform hyperstar and the k-uniform hyperstar. Some of the recurrence relations are derived for some of the coefficients of characteristic equation.

The general form of an n degree characteristic equation is taken as $-\lambda^n + C_{n-1}\lambda^{n-1}+$

$C_{n-2}\lambda^{n-2} + \cdots + C_3\lambda^3 + C_2\lambda^2 + C_1\lambda + C_0$. For a k-uniform hyperstar, the number of vertices is given by the form, $n = (m+1)k - m + k - 1$; $m = 1, 2, 3, ...$ The sum of the product of two eigen values of a k-uniform hyperstar is given by $\sum_{i,j=1}^{(m+1)k-m+k-1} \lambda_i\lambda_j = \frac{k}{2}[(m+2)k-m-2]$ or $P[(m+1)k-m+k-1] = \frac{k}{2}[(m+2)k-m-2]$, where $m = 1, 2, 3 \cdots$ with the recurrence relation $P[(m+1)k-m+k-1] = P[(m+1)k-m] + \frac{k(k-1)}{2}$; $P(2k-1) = k(k-1)$. The function $s(j) = \sum_{i=1}^{j} \frac{i(i+1)}{2}$ is defined for proving the following statement. The sum of the product of three eigenvalues of a k-uniform hyperstar is given by $\sum_{i,j,k=1}^{(m+1)k-m+k-1} \lambda_i\lambda_j\lambda_k = s(k-2)\ 2(m+1)$; , or $P[(m+1)k-m+k-1] = s(k-2)\ 2(m+2)$, where $m = 1, 2, 3 \cdots$ with the recurrence relation $P[(m+1)k-m+k-1] = P[(m+1)k-m] + 2\ s(k-2)$; $P(2k-1) = 4\ s(k-2)$

C48

Coefficients of characteristic polynomial of skew Laplacian matrix of some directed graphs

Archana Ingole[1] and Ujawala Deshmukh[2]

[1]Pillai College of Engineering, New Panvel, India

[2]Mithibhai College,Vileparle Mumbai, India

In this paper we give combinatorial expression for characteristic coefficients of skew Laplacian matrix of digraphs.For path we have derived seventh coefficient,for cycle we attempted to calculate general coefficient,and for star with inward direction as well as outward direction derived general characteristic coefficient of its skew Laplacian matrix.

Some properties of extended transformation graph of a graph C50

Manali Sharma and Pravin Garg

Department of Mathematics, University of Rajasthan, Jaipur, India

The extended transformation graph $\mathbb{G}_{\mathrm{e}}{}^{+++}_{+++}(G)$ of a graph $G = (V, E)$ is the graph with vertex set $V \cup E \cup \overline{E}$, in which two vertices are adjacent if the corresponding vertices, edges or non-edges are adjacent or incident in G. The extended transformation graph $\mathbb{G}_{\mathrm{e}}{}^{+-+}_{---}(G)$ of a graph $G = (V, E)$ is the graph with vertex set $V \cup E \cup \overline{E}$, in which two vertices v_i and v_j are adjacent if and only if (i) $v_i, v_j \in V$ and v_i, v_j are adjacent in G, (ii) $v_i, v_j \in E$ and v_i, v_j are not adjacent in G, (iii) $v_i \in V, v_j \in E$ and v_j is incident at v_i in G, (iv) $v_i, v_j \in \overline{E}$ and v_i, v_j are not adjacent in $\overline{G}$, (v) $v_i \in V, v_j \in \overline{E}$ and v_j is not incident at v_i in $\overline{G}$, (vi) $v_i \in E, v_j \in \overline{E}$ and v_i, v_j are not adjacent in K_n In this paper, we study some properties of the extended transformation graph $\mathbb{G}_{\mathrm{e}}{}^{+++}_{+++}(G)$ and the extended transformation graph $\mathbb{G}_{\mathrm{e}}{}^{+-+}_{---}(G)$.

Role of typed lambda calculus λ in analyzing natural language expressions in semantics C52

Harjit Singh

Indira Gandhi National Tribal University, India

The present study focuses on the general survey on lambda abstraction λ in a natural language. Lambda λ is essential application used in mathematics, computer science, logic and semantics. We have tried to find out the formal representation for natural language expressions through lambda λ particularly in semantics and compute 1:1 correspondence between two different natural languages. Based on a lambda λ computation, we able to propose an algorithm in analysing single and complex propositions and planning to include double complex and compound predicates, reduplicated forms and mainly addressing notes are typical instances in future research.

Keywords: Lambda operator λ, natural language, semantics, syntax

C53 Mycielskian of graphs with small game domination number

Tijo James

Department of Mathematics, Pavanatma College, Kerala, India

Domination game is a game played on a finite, undirected graph G, between two players Dominator and Staller. During the game, the players alternately choose vertices of G such that each chosen vertex dominates at least one new vertex that is not dominated by previously chosen vertices. The aim of Dominator is to finish the game as early as possible while that of Staller is to delay the process as much as possible. The game domination number $\gamma_g(G)$ is the number of moves of both players when Dominator starts the game and the staller start game domination number $\gamma_g'(G)$ is the number of moves of both players when Staller starts the game.

Here the domination game in the Mycielskian of a graph is studied. We establish bounds for the game domination number of Mycielskian of a graph in terms of its domination number and also in terms of its game domination number. The Mycielskian of a graph with small game domination number are characterised.

C54 Splitters and decomposers for binary matroids

Sandra Kingan

Brooklyn College, City University of New York, USA

Let $\mathcal{M}$ denote the class of binary matroids with no minors isomorphic to $M_1, \ldots, M_k$. A splitter for $\mathcal{M}$ is a 3-connected matroid N in $\mathcal{M}$ such that no 3-connected matroid in $\mathcal{M}$ has a proper N-minor. A 3-decomposer for $\mathcal{M}$ is a 3-connected matroid N in $\mathcal{M}$ with a non-minimal exact 3-separation (A, B) such that any matroid M in $\mathcal{M}$ with an N-minor has a 3-separation (X, Y) such that $A \subset X$ and $B \subset Y$. Splitters and 3-decomposers capture the structure present in excluded minor classes. In this paper we give a decomposition theorem for binary matroids with no minors isomorphic to S_{10} or S_{10}^*, where S_{10} is a certain 10-element rank-4 matroid. As a corollary we obtain a decomposition result for the class of binary matroids with no minors isomorphic to $M(K_{3,3})$ or $M^*(K_{3,3})$. These two results imply a 2004 result on internally 4-connected matroids by Zhou (JCTB 91, 327-343) and a 2017 result by Mayhew, Royle, and Whittle (JCTB 125, 95–113).

S_4-decomposition of the line graph of the complete graph C56

K. Arthi, R. Sangeetha and C. Sankari

Department of Mathematics, A. V. V. M. Sri Pushpam College (Affiliated to Bharathidasan University), India

Let S_k denote a star with k edges. The line graph of the complete graph K_n is denoted by $L(K_n)$. In this paper, we prove that the graph $L(K_n)$ has an S_4-decomposition if and only if $n \geq 6$ and $n \equiv 0, 1, 2, 4, 6 (mod\ 8)$.

Connectivity of the tensor product of graphs and cycles C57

A.V. Sonawane[1] and Y. M. Borse[2]

[1]Government of Maharashtra's Ismail Yusuf College of Arts, Science and Commerce, India

[2]Department of Mathematics, Savitribai Phule Pune University, India

In this paper, we determine the connectivity of the tensor product $G \times C$ of a graph G and a cycle C. We get exact formula for the connectivity of $G \times C$ when either G is bipartite or C is even cycle, and obtain upper and lower bounds when G is non-bipartite and C is odd cycle. As a consequence, we prove that the connectivity of the tensor product of k odd cycles is 2^k.

C58 **Properties of semi splitting block graph of a graphs**

Nivedha Baskar, Tabitha Agnes Mangam and Mukti Acharya

Department of Mathematics, CHRIST(Deemed to be University), Bangalore, India

For any graph G, the semi splitting block graph $S_B(G)$ of a graph is obtained by taking a copy of G, for each vertex in G a new vertex is added which is made adjacent to all the vertices of G adjacent to that vertex and for each block in G a new vertex is added which is made adjacent to all the vertices of that block. We study the structural properties of semi splitting block graph of certain classes of graphs. The connectivity parameters and invariants of covering number of semi splitting block graph of a graph are investigated. We also proved that for any connected graph, $S_B(G)$ is never Eulerian.

C59 **Proofs of Berkovich and Uncu's conjectures on integer partitions using Frobenius numbers**

Damanvir Singh Binner[1] and Amarpreet Rattan[2]

Simon Fraser University, Canada

We use techniques from elementary number theory (such as Frobenius numbers) to combinatorially prove four recent conjectures of Berkovich and Uncu (Ann. Comb. 23 (2019) 263–284) regarding inequalities between the sizes of two closely related sets consisting of integer partitions whose parts lie in the interval $\{s, ..., L + s\}$. Further restrictions are placed on the sets by specifying impermissible parts as well as a minimum part.

Keywords: Integer Partitions, Frobenius Numbers, Combinatorial Proofs, Explicit Bounds

---

Center of cartesian and strong product of digraphs C60

Mary Shalet Thottungal Joseph and Prasanth G. Narasimha-Shenoi

Department of Mathematics, Government College, Chittur, India

Let $D = (V, E)$ be a digraph and $u, v \in V(D)$. The metric maximum distance is defined by $md(u, v) = \max\{\overrightarrow{d}(u, v), \overrightarrow{d}(v, u)\}$ where $\overrightarrow{d}(u, v)$ denote the length of a shortest directed $u - v$ path in D. The center $C(D)$ of a strongly connected digraph consist of all the vertices with minimum eccentricity. The relationship between the center of the Cartesian and strong product of two or more digraphs and its factor graphs have been studied in this article.

Frankl's conjecture and lattices derived from reduced semigroups C61

Vinayak Joshi

Department of Mathematics, Savitribai Phule Pune University, India

In this paper, we prove Frankl's Conjecture for the class of lattices (a subclass of 0-distributive lattices) derived from reduced semigroups, which satisfies the annihilator condition. Also, we observe that the class of multiplicative lattices also satisfies Frankl's Conjecture.

Disjoint cycles through prescribed vertices in multidimensional tori C64

Amruta Shinde and Y. M. Borse

Department of Mathematics, Savitribai Phule Pune University, India

A graph G is spanning r-cyclable if for any r distinct vertices $v_1, v_2, \ldots, v_r$, there exists r vertex-disjoint cycles of G such that union of these cycles spans G and each cycle contains exactly one v_i for $1 \leq i \leq r$. It is known that the hypercube Q_n and its variation the crossed cube CQ_n are r-cyclable for $1 \leq r \leq n-1$. We prove that the n-dimensional torus, different from $C_3 \square C_3$, is spanning r-cyclable for $1 \leq r \leq 2n - 1$.

C67 **On the intersection ideal graph of semigroups**

Jitender Kumar and Barkha

epartment of Mathematics, Birla Institute of Technology and Science Pilani, India

The intersection ideal graph $\Gamma(S)$ of a semigroup S is a simple undirected graph whose vertices are all nontrivial left ideals of S and two distinct left ideals I, J are adjacent if and only if their intersection is nontrivial. In this paper, we investigate the connectedness of $\Gamma(S)$. We show that if $\Gamma(S)$ is connected then $diam(\Gamma(S)) \leq 2$. Further we classify the semigroups such that the diameter of their intersection graph is two. Other graph invariants, namely perfectness, planarity, girth, dominance number, clique number etc. are also discussed. Finally, if S is union of n minimal left ideals then we obtain the independence number, metric dimension and automorphism group of $\Gamma(S)$.

C69 **On coupon coloring of Cayley graphs**

Pavithra R and Reji T

Government College, Chittur, Kerala, India

A k- coupon coloring of a graph G without isolated vertices is an assignment of colors from $[k] = \{1, 2, \ldots, k\}$ to the vertices of G such that the neighborhood of every vertex of G contains vertices of all colors from $[k]$. The maximum k for which a k-coupon coloring exists is called the coupon coloring number of G. The Cayley Graph $Cay(G, C)$ of a group G is a graph with vertex set G and edge set $E(Cay(G, C)) = \{gh : hg^{-1} \in C\}$, where C is a subset of G that is closed under taking inverses and does not contain the identity. For a commutative ring R with unity, $Cay(R^+, Z(R)^*)$ is denoted by $\mathbb{C}AY(R)$, where R^+ is the additive group and $Z(R)^*$ is the nonzero zero-divisors of R. In this paper, we have obtained bounds for the coupon coloring number of $\mathbb{C}AY(\mathbb{Z}_n)$ and $\mathbb{C}AY(\mathbb{Z}_n \times \mathbb{Z}_m)$, where $\mathbb{Z}_n$ is the commutative ring of integers modulo n and $\mathbb{Z}_n \times \mathbb{Z}_m$ is the Cartesian product of $\mathbb{Z}_n$ and $\mathbb{Z}_m$. We also found that in some cases these upper bounds are sharp. We have

found the coupon coloring number of $Cay(\mathbb{Z}_n, C)$ when $C = \{1, -1, a = -a\}$ and $C = \{1, -1, 2, -2\}$.

Fold thickness of graphs C70

Vaishnavi S[1] and Reji T[2]

[1]Sree Narayana College, Palakkad, Kerala, India

[2]Government College, Chittur, Kerala, India

The graph G' obtained from a graph G by identifying two nonadjacent vertices in G having at least one common neighbor and reducing the resulting multiple edges to simple edges is called a 1-fold of G. A uniform k-folding of a graph G is a sequence of graphs $G = G_0, G_1, G_2, \ldots, G_k$, where G_{i+1} is a 1-fold of G_i for $i = 0, 1, 2, \ldots, k-1$ such that all graphs in the sequence are singular or all of them are nonsingular. The largest k for which there exists a uniform k- folding of G is called fold thickness of G and this concept was first introduced by Gervacio et al. In this paper, we determine fold thickness of lollipop graph, web graph, helm graph and rooted product of complete graphs and paths.

Restricted shuffle of Parikh word representable graphs C71

Nobin Thomas[1], Lisa Mathew[1], Somnath Bera[2] and K.G. Subramanian

[1]Amal Jyothi College of Engineering, Kanjirappally, Kerala, India

[2]School of Advanced Sciences-Mathematics, Vellore Institute of Technology, Chennai, Tamil Nadu, India

[3]Department of Mathematics and Computer Science, Faculty of Science, Liverpool Hope University, Liverpool, UK

A number of studies linking words and graphs have appeared recently. In 2016, Bera and Mahalingam introduced a graph, called the Parikh word representable graph, associated with a word w based on the scattered subwords of length two involving consecutive symbols in an ordered alphabet. On the other hand the restricted shuffle operation has been studied extensively. Here we derive several word

properties and graph properties of the Parikh word representable graph of a restricted shuffle of words studied by Atanasiu and Teh.

C75

Domination number of a complement of signed graph

P. Jeyalakshmi[1], K. Karuppasamy[1] and S. Arockiaraj[2]

[1]Department of Mathematics, Kalasalingam Academy of Research and Education, Krishnankoil, India

[2]Department of Mathematics, Government Arts and Science college, Sivakasi, Tamil Nadu, India

From a signed graph Σ, a signed graph Σ' is obtained whose vertex set is as in Σ and the edge set is obtained by the way of obtaining the edges for the complement of graph. Since any two vertices of Σ is non adjacent or adjacent with positive or negative sign, the edges of Σ' may be the combination of negation of these three. Among these combination, the complement of first kind of Σ say Σ^c_α as follows. If uv is a negative edge in Σ, then u and v are assumed to be non-adjacent in Σ'. If uv is a positive edge in Σ, then it is assumed to be a negative edge in Σ'. If u and v are non-adjacent in Σ, then uv is an edge with positive sign in Σ'. In this paper, we obtained some bounds for domination number of Σ^c_α. Further more some Nordhaus - Gaddum type results are obtained.

Comparability graphs which are cover-incomparability graphs C83

Arun Anil and Manoj Changat

Department of Futures Studies, University of Kerala, India.

Comparability graphs and cover-incomparability graphs (C-I graphs) are two interesting classes of graphs from posets. Comparability graph of a poset $P = (V, \leq)$ is a graph with vertex set V and two vertices u and v is adjacent in V if u and v are comparable in P. A C-I graph is a graph from P with vertex set V and the edge-set is the union of edge-sets of the cover graph and the incomparability graph of the poset. C-I graphs have interesting implications on both graphs and posets. In this paper, the C-I graphs which are also comparability graphs are studied and identify the class of C-I graphs which are Ptolemaic, threshold graphs and bisplit graphs (all these graphs are subclasses of comparability graphs). Finally, we prove that the family of split graphs which are C-I graphs are also comparability graphs.

An exact formula for a Lambert series associated to a cusp form and the Möbius function C86

Abhishek Juyal[1], Bibekananda Maji[2] and Sumukha Sathyanarayana[3]

[1]Department of Mathematics, The Institute of Mathematical Sciences, India

[2]Discipline of Mathematics, Indian Institute of Technology Indore, India

[3]Department of Mathematical and Computational Sciences, National Institute of Technology Karnataka, Suratkal, India

In 1981, Zagier conjectured that the constant term of the automorphic form $y^{12}|\Delta(z)|^2$, that is, $a_0(y) := y^{12}\sum_{n=1}^{\infty}\tau^2(n)\exp(-4\pi ny)$, where $\tau(n)$ is the nth Fourier coefficient of the Ramanujan cusp form $\Delta(z)$, has an asymptotic expansion when $y \to 0^+$ and it can be expressed in terms of the non-trivial zeros of the Riemann zeta function $\zeta(s)$. This conjecture was settled by Hafner and Stopple, and later Chakraborty, Kanemitsu, and Maji extended this result for any Hecke eigen form over the full modular group. In this talk, we will investigate a Lambert

series associated to the Fourier coefficients of a cusp form and the Möbius function $\mu(n)$. We will present an exact formula for the Lambert series and we see that, the main term has been expressed in terms of the non-trivial zeros of $\zeta(s)$, and the error term expressed as an infinite series involving generalized hypergeometric series ${}_2F_1(a, b; c; z)$. As an application of this exact form, we will also establish an asymptotic expansion of the Lambert series. This is joint work with Abhishek Juyal and Bibekananda Maji.

C88

Generalization of five q-series identities of Ramanujan and unexplored weighted partition identities

Subhash Chand Bhoria[1], Pramod Eyyunni[2] and Bibekananda Maji[23]

[1]Pt. Chiranji Lal Sharma Government PG College, Karnal, Haryana, India

[2]Indian Institute of Science Education and Research Berhampur, India

[3]Discipline of Mathematics, Indian Institute of Technology Indore, India

The unorganized portion of Ramanujan's second notebook contains five interesting q-series identities. Recently, in joint work with A. Dixit, we found a one-variable generalization of one of the aforementioned five identities of Ramanujan and derived the last three q-series identities, but couldn't connect with the first two identities. In this work, we establish a new q-series identity from which we successfully deduce all the five q-series identities of Ramanujan and also derive a few interesting weighted partition identities. This is joint work with S. Chand and P. Eyyunni.

Author Index